

# Abductive Inference and Epistemic Closure in Christian Apologetics

Phil Stilwell

Independent Scholar

*Thesis:* Abductive inference—inference to the best explanation (IBE)—cannot by itself justifiably sustain high credence in Christian miracle claims, and attempts to close belief under abduction-driven entailments (“epistemic closure” in apologetic cumulative cases) fail once we properly model low priors, dependence among testimonies, and unconceived alternatives. We motivate the thesis with the historical shift from supernatural attributions of lightning to atmospheric electricity (Beard & Henderson, 2001; Uman, 2001); integrate critiques of abduction’s domain-dependence and the problem of unconceived alternatives (Norton, 2019; Stanford, 2006; van Fraassen, 1980); analyze the misuse of closure principles when moving from generic theism to a specific miracle (Dretske, 1970; Hawthorne, 2004; Nozick, 1981; Pollock, 1986); and appraise resurrection-centered abductive arguments under realistic priors and likelihoods that reflect base-rate scarcity and testimonial dependence (Earman, 2000; Fitelson, 2007; Licona, 2010; McGrew & McGrew, 2009; Sober, 2008; Wright, 2003). We conclude with a gradient epistemology: credences should scale with discriminative, testable evidence rather than with explanatory neatness alone.

## **Opening Narrative: From Zeus to Electrons**

For much of human history, lightning was terrifying and mysterious. Greeks spoke of Zeus; other cultures posited Thor or Raijin. These attributions were not mere fables; given a narrow hypothesis space, divine agency seemed the “best explanation” (Beard &

Henderson, 2001). The maturation of atmospheric physics reframed the explanandum: charge separation, stepped leaders, and return strokes now explain lightning as a high-voltage discharge (Uman, 2001).

The lesson is epistemic, not merely historical. IBE is a powerful *heuristic* for hypothesis generation (Douven, 2017; Lipton, 2004), but it is unreliable where (i) background theories are immature, (ii) alternatives are unconceived (Stanford, 2006), and (iii) explanatory virtues poorly track truth (Norton, 2019; van Fraassen, 1980). When the hypothesis space widens, yesterday’s “best” often becomes false.

### **Core Framework: Abduction Within a Bayesian Partition**

Let  $H$  be a target hypothesis (e.g., “Jesus was bodily resurrected”), and  $E$  be the evidence (e.g., post-mortem appearance traditions). Bayes’ theorem:

$$P(H | E) = \frac{P(E | H) P(H)}{P(E | H) P(H) + P(E | \neg H) P(\neg H)}. \quad (1)$$

*This displays the posterior as the prior times a likelihood-ratio penalty/boost, normalized against  $\neg H$ .*

Partition  $\neg H$  into specified rivals  $R_1, \dots, R_n$  and an “unknowns” class  $U$  (*unconceived alternatives*):

$$P(H | E) = \frac{P(E | H) P(H)}{P(E | H) P(H) + \sum_{i=1}^n P(E | R_i) P(R_i) + P(E | U) P(U)}. \quad (2)$$

*Any nonzero mass on  $U$  lowers  $P(H | E)$  unless  $P(E | U)$  is near zero; this formalizes the unconceived-alternatives problem (Stanford, 2006).*

Because miracle claims have extreme base-rate scarcity,  $P(H)$  is tiny absent antecedent theism; testimonial  $E$  is often dependence-prone, which inflates  $P(E | \neg H)$  (Earman, 2000; Sober, 2008). Thus, even apparently high “explanatory fit” cannot overcome realistic priors when denominators include  $U$ .

**Bridge to Closure.** The preceding Bayesian picture shows *why* abductive neatness and even sizable likelihood ratios need not deliver high posteriors. Low priors, dependence, and  $U$  keep the denominator large. Faced with this, apologetic strategies often shift from local confirmation to a *global* move: if IBE supports generic theism, perhaps belief in a specific miracle is secured by *epistemic closure*. The next section explains why this closure move fails.

### Epistemic Closure and the Apologist's "Cumulative Case"

A familiar apologetic pattern is: (1) abduction supports God's existence; (2) theism raises the antecedent probability of resurrection; therefore (3) abductively the resurrection occurred. This tacitly treats credences as closed under known implication from theism to miracle.

In epistemology, closure is contested: classical closure ( $Kp$  and  $K(p \rightarrow q)$  entail  $Kq$ ) was challenged by Dretske (1970) and Nozick (1981), who deny that knowledge necessarily closes under known entailment in cases with relevant alternatives or tracking conditions. Even on closure-friendly views, closure is *defeasible* when underminers arise (Hawthorne, 2004). For *credences*, an analogous defeasibility holds: abductive support for a generic theism does not *close* to high credence in a particular miracle when there are live defeaters (low base rates, testimonial dependence, sociological mechanisms). Pollock's undercutting-defeaters framework makes precise how such information defeats prima facie justification (Pollock, 1986). In short, cumulative cases typically *stack defeasible supports* rather than transmit warrant via closure; the final step inherits the weakest link.

**Plantinga and Warrant.** A further apologetic line appeals not to Bayesian confirmation but to Plantinga (2000), who argues that belief in the resurrection may be "properly basic," warranted apart from evidential calibration. Our analysis highlights that even if such warrant is conceded on internalist grounds, it does not alter the external

evidential landscape. Bayesian deficits remain: low priors, dependence, and unconceived alternatives ensure that abduction plus closure cannot secure high public credence.

Plantinga's account therefore sits orthogonally to evidential evaluation; it offers a theory of private warrant but not a defeater for probabilistic critiques.

### **Resurrection as a Test Case: Likelihoods Under Realistic Priors**

Consider stylized scenarios, chosen to be friendly to  $H$  yet still realistic.

#### **Scenario R-1: Friendly priors, no unknowns**

$$P(H) = 10^{-2}, \quad P(E | H) = 0.8, \quad P(E | \neg H) = 0.10,$$

$$P(H | E) = \frac{0.8 \times 10^{-2}}{0.8 \times 10^{-2} + 0.10 \times 0.99} \approx 0.075.$$

*Even with optimistic inputs, the posterior is only  $\sim 7.5\%$ .*

#### **Scenario R-2: Neutral prior, unknowns included**

$$P(H) = 10^{-6}, \quad P(U) = 0.80, \quad P(R) = 0.20,$$

$$P(E | H) = 0.7, \quad P(E | R) = 0.05, \quad P(E | U) = 0.10,$$

$$P(H | E) \approx \frac{0.7 \times 10^{-6}}{0.7 \times 10^{-6} + 0.05 \times 0.20 + 0.10 \times 0.80} \approx 7.8 \times 10^{-6}.$$

*Allowing for unconceived alternatives keeps the posterior near the tiny prior.*

#### **Scenario R-3: Optimistic LR for $H$**

$$P(H) = 10^{-6}, \quad P(E | H) = 0.95, \quad P(E | \neg H) = 0.02,$$

$$P(H | E) \approx \frac{0.95 \times 10^{-6}}{0.95 \times 10^{-6} + 0.02 \times (1 - 10^{-6})} \approx 4.8 \times 10^{-5}.$$

*Even a large likelihood ratio cannot surmount a microscopic prior without overwhelming, independent evidence.*

These results cohere with Humean-Bayesian analyses of testimonial evidence for class-violating events (Earman, 2000; Sober, 2008) and with formal work showing that explanatory power does not guarantee confirmational force absent appropriate priors (Fitelson, 2007; Schupbach & Sprenger, 2011).

**Transition to General Lessons.** Having seen that even generous Bayesian inputs fail to yield high posteriors, we now broaden to the epistemic role of abduction in general. The next section shows that abduction retains heuristic value in mature science even where it falters in apologetic domains.

### **Abduction's Value and Its Limits**

Abduction remains indispensable for *generating* hypotheses and guiding attention (Douven, 2017; Lipton, 2004). But its reliability is *domain-dependent*: where background theory is mature and tests discriminate rivals, explanatory virtues better track truth; where background theory is thin and tests are unavailable (e.g., singular miracle claims), IBE overreaches (Norton, 2019; Stanford, 2006; van Fraassen, 1980). The historical demotion of gods-of-the-gaps explanations (Zeus, miasma, phlogiston) illustrates the point (Gleiser, 2014; Kuhn, 1962; Porter, 1997).

### **Counterarguments and Replies**

#### **“Cumulative-Case” Bayes for the Resurrection**

Advocates argue that many moderate strands of evidence multiply to a strong posterior (Licona, 2010; McGrew & McGrew, 2009; Wright, 2003). Reply: (i) dependence among testimonies and traditions reduces multiplicativity; (ii) low base rates keep priors small; (iii) unconceived alternatives ( $U$ ) claim nontrivial prior mass; (iv) likelihoods for  $R_i$  and  $U$  are not negligible once sociological, cognitive, and textual mechanisms are

considered. Aggregation therefore often yields only modest *LRs*, insufficient to overcome the prior (Earman, 2000; Fitelson, 2007; Sober, 2008).

### **“IBE Is Respectable in Science; Why Not Here?”**

True: IBE is integral to scientific practice (Douven, 2017; Lipton, 2004). But reliability tracks domain maturity and testability; where experiments can discriminate rivals, explanatory virtues correlate with truth. Singular miracle claims lack such discrimination, so the *transfer* from scientific IBE to apologetic IBE is a category mistake (Norton, 2019; Stanford, 2006; van Fraassen, 1980).

### **“Closure from Theism to Miracle”**

Even granting abductive support for generic theism, closure to a specific miracle is blocked by live defeaters and relevant alternatives. Closure is at best defeasible for knowledge (Dretske, 1970; Hawthorne, 2004; Nozick, 1981) and a fortiori for credences. Pollock-style undercutters show how information about dependence, error, and base rates undercuts the transmission of warrant in cumulative chains (Pollock, 1986).

**Transition to Conclusion.** These objections, once addressed, reinforce the central lesson: abduction can guide inquiry but does not close belief over extraordinary claims. The conclusion articulates this lesson as a call for epistemic humility.

### **Conclusion: A Gradient Epistemology**

A rational stance uses a *gradient epistemology*: credences track the tested balance of evidence and fall when defeaters accumulate. Abduction should launch inquiry, not close it. In apologetics, treating IBE as warrant-transmitting closure—from God to specific miracle—ignores low priors, dependence, and unconceived alternatives. Historically and formally, such overreach recapitulates the Zeus-to-electrons trajectory. The responsible posture is probabilistic humility: let beliefs scale with reproducible tests and discriminative evidence.

### References

- Beard, M., & Henderson, J. (2001). *Classical art: From greece to rome*. Oxford University Press.
- Douven, I. (2017). Inference to the best explanation: What is it and why should we care? In K. McCain & T. Poston (Eds.), *Best explanations: New essays on inference to the best explanation* (pp. 1–24). Oxford University Press.
- Dretske, F. I. (1970). Epistemic operators. *The Journal of Philosophy*, 67(24), 1007–1023. <https://doi.org/10.2307/2024713>
- Earman, J. (2000). *Hume's abject failure: The argument against miracles*. Oxford University Press.
- Fitelson, B. (2007). Likelihoodism, bayesianism, and relational confirmation. *Synthese*, 156(3), 473–489. <https://doi.org/10.1007/s11229-006-9136-9>
- Gleiser, M. (2014). *The island of knowledge: The limits of science and the search for meaning*. Basic Books.
- Hawthorne, J. (2004). *Knowledge and lotteries*. Oxford University Press.
- Kuhn, T. S. (1962). *The structure of scientific revolutions*. University of Chicago Press.
- Licona, M. R. (2010). *The resurrection of jesus: A new historiographical approach*. IVP Academic.
- Lipton, P. (2004). *Inference to the best explanation* (2nd). Routledge.
- McGrew, T., & McGrew, L. (2009). The argument from miracles: A cumulative case for the resurrection of jesus of nazareth. In W. L. Craig, J. P. Moreland, & C. Meister (Eds.), *The blackwell companion to natural theology* (pp. 593–662). Wiley-Blackwell.
- Norton, J. D. (2019). *The material theory of induction*. Cambridge University Press.
- Nozick, R. (1981). *Philosophical explanations*. Harvard University Press.
- Plantinga, A. (2000). *Warranted christian belief*. Oxford University Press.

- Pollock, J. L. (1986). *Contemporary theories of knowledge*. Rowman & Littlefield.
- Porter, R. (1997). *The greatest benefit to mankind: A medical history of humanity*. W. W. Norton & Company.
- Schupbach, J. N., & Sprenger, J. (2011). The logic of explanatory power. *Philosophy of Science*, 78(1), 105–127. <https://doi.org/10.1086/658111>
- Sober, E. (2008). *Evidence and evolution: The logic behind the science*. Cambridge University Press.
- Stanford, K. (2006). *Exceeding our grasp: Science, history, and the problem of unconceived alternatives*. Oxford University Press.
- Uman, M. A. (2001). *The lightning discharge* (2nd). Dover.
- van Fraassen, B. C. (1980). *The scientific image*. Oxford University Press.
- Wright, N. T. (2003). *The resurrection of the son of god*. Fortress Press.

## Appendix A

### Appendix A: Bayesian Skeleton for Abductive Overreach

**Partition Law.**

$$P(E) = P(E | H)P(H) + \sum_{i=1}^n P(E | R_i)P(R_i) + P(E | U)P(U). \quad (\text{A1})$$

*Evidence is a mixture over mutually exclusive and exhaustive rivals, including unknowns  $U$ .*

**Mixture Likelihood and Specificity Penalty.**

$$P(H | E) = \frac{P(E | H)P(H)}{P(E)} \Rightarrow \text{as } P(U) \uparrow \text{ with } P(E | U) > 0, P(H | E) \downarrow. \quad (\text{A2})$$

*Unconceived alternatives impose a specificity penalty on  $H$ :  $E$  is less specific to  $H$  when many ways make  $E$  unsurprising.*

**Dependence Correction.**

$$P(E_1 \wedge \dots \wedge E_m | \neg H) \geq \prod_{j=1}^m P(E_j | \neg H). \quad (\text{A3})$$

*Testimonial or tradition dependence inflates the joint likelihood under  $\neg H$ , reducing the overall LR. Assuming independence overstates support.*

**Likelihood Flatness.**

$$\text{If } P(E | R_i) \approx P(E | R_k) \approx P(E | U), \quad (\text{A4})$$

*then no single rival earns a decisive LR advantage; the posterior remains prior-dominated.*

## Appendix B

### Appendix B: Symbolic and Syllogistic Forms (Expanded Gloss)

#### S1: Abduction Is Non-Closing for Miracles

**P1** (Base-rate scarcity)  $P(\text{bodily resurrection}) \ll 1$  absent strong theism.

**P2** (Testimony dependence) Most  $E$  is socially dependent.

**P3** (Unknowns)  $P(U) > 0$  and  $P(E | U) > 0$ .

**C** Therefore, IBE does not transmit high credence to specific miracles.

*Gloss:* P1 encodes the rarity of class-violating events; P2 notes that correlated sources raise  $P(E | \neg H)$ ; P3 ensures that unexplored rivals keep the denominator large, preventing closure from IBE to belief.

#### S2: Closure Blocked by Undercutters

**P1** If there are live relevant alternatives or undercutters, closure fails to transmit warrant (Dretske, 1970; Nozick, 1981; Pollock, 1986).

**P2** Miracle inferences face relevant naturalistic alternatives and undercutters.

**C** Therefore, abductive warrant for theism does not close to warrant for a specific miracle.

*Gloss:* Closure requires stability under known implications; undercutters (e.g., dependence, error mechanisms) defeat stability, so warrant does not transmit to the miracle claim.

**Fitch-Style Sketch (Readable Notes)**

1.  $E$  *(evidence: appearances/traditions)*
2.  $H$  *(target: resurrection)*
3.  $\neg H \equiv \bigvee_{i=1}^n R_i \vee U$  *(partition adds unconceived rivals)*
4.  $LR = \frac{P(E | H)}{P(E | \neg H)}$  *(explanatory power as a likelihood ratio)*
5.  $P(\neg H) \gg P(H)$  *(base-rate scarcity for miracles)*
6.  $P(E | \neg H)$  high via dependence *(correlated testimony boosts the denominator)*
7.  $P(U) > 0 \wedge P(E | U) > 0$  *(unknown mechanisms can also produce E)*
8.  $\therefore P(H | E)$  small *(posterior remains low despite IBE neatness)*

### Appendix C

#### Appendix C: Contemporary Testimonial Case (Heuristic Analysis)

**Case.** Multiple congregants report a sudden healing during prayer. Reports are temporally clustered and partially overlapping.

**Model sketch.** Let  $H$  = “a class-violating healing occurred.” Rivals include  $R_1$  =misdiagnosis,  $R_2$  =natural remission,  $R_3$  =reporting/selection effects,  $R_4$  =memory conformity, and  $U$  =unconceived mechanisms. With  $m$  testimonies, dependence parameter  $\rho \in [0, 1]$  adjusts the joint likelihood under  $\neg H$ :

$$P(E_1 \wedge \dots \wedge E_m \mid \neg H, \rho) \approx (\bar{p})^{m(1-\rho)} \cdot \bar{p}^\rho,$$

where  $\bar{p}$  is the marginal  $P(E_j \mid \neg H)$ . Higher  $\rho$  inflates the joint probability under  $\neg H$ .

*Example.* Suppose three congregants independently have a 70% chance of reporting an apparent healing under natural mechanisms. If their testimonies are fully independent ( $\rho = 0$ ), the joint probability under  $\neg H$  is about  $0.7^3 \approx 0.34$ . If dependence is high ( $\rho = 0.5$ ), the joint probability rises closer to 0.7, meaning the three reports are epistemically no stronger than a single semi-independent report. This illustrates how dependence erodes the cumulative force of testimonial evidence.

**Symbolic Logic Formalization**

$E(x)$  :  $x$  involves evidence testing and adaptability  
**Annotation:** Define  $E(x)$  to mean that reasoning method  $x$  relies on testing evidence and allows conclusions to change.

$S(x)$  :  $x$  is superior for rational decision-making  
**Annotation:** Define  $S(x)$  to mean that reasoning method  $x$  counts as a superior approach for rational conclusions.

$P(x)$  :  $x$  leads to premature conclusions without verification  
**Annotation:** Define  $P(x)$  to mean that reasoning method  $x$  produces premature conclusions without sufficient verification.

$L(x)$  :  $x$  is limited and not superior for rational decision-making  
**Annotation:** Define  $L(x)$  to mean that reasoning method  $x$  is epistemically limited and cannot be considered superior.

Premises about Induction  
 $\forall x [E(x) \rightarrow S(x)]$   
**Annotation:** For all reasoning methods, if a method involves evidence testing and adaptability, then it is superior for rational decision-making.

$E(I)$   
**Annotation:** Inductive reasoning (I) involves evidence testing and adaptability.

Conclusion from Induction  
 $S(I)$   
**Annotation:** Therefore, inductive reasoning (I) is superior for rational decision-making.

Premises about Abduction  
 $\forall x [P(x) \rightarrow L(x)]$   
**Annotation:** For all reasoning methods, if a method leads to premature conclusions without verification, then it is limited and not superior for rational decision-making.

$P(A)$   
**Annotation:** Abductive reasoning (A) leads to premature conclusions without thorough verification.

Conclusion from Abduction  
 $L(A)$   
**Annotation:** Therefore, abductive reasoning (A) is limited and not superior for rational decision-making.

---

**An expanded syllogistic chain version.**

**1) Vocabulary and constants**

$E(x)$  :  $x$  involves evidence testing and adaptability  
**Annotation:** Predicate  $E(x)$  says that a method  $x$  systematically tests evidence and allows revision.

$S(x)$  :  $x$  is superior for rational decision-making  
**Annotation:** Predicate  $S(x)$  marks method  $x$  as rationally superior.

$P(x)$  :  $x$  leads to premature conclusions without thorough verification  
**Annotation:** Predicate  $P(x)$  says method  $x$  tends to close inquiry too early.

$L(x)$  :  $x$  is limited and not superior for rational decision-making  
**Annotation:** Predicate  $L(x)$  says method  $x$  is not rationally superior.

$I$  : the method of induction  
**Annotation:** Constant  $I$  denotes inductive reasoning.

$A$  : the method of abduction (IBE)  
**Annotation:** Constant  $A$  denotes abductive reasoning (inference to the best explanation).

---

**2) General axioms (method criteria)**

$\forall x (E(x) \rightarrow S(x))$   
**Annotation:** Any method with property  $E$  (evidence-testing, adaptable) is rationally superior  $S$ .

$\forall x (P(x) \rightarrow L(x))$   
**Annotation:** Any method with property  $P$  (premature closure) is limited  $L$ .

---

**3) Factual premises about specific methods**

$E(I)$   
**Annotation:** Induction  $I$  involves evidence-testing and adaptability.

$P(A)$   
**Annotation:** Abduction  $A$  (when treated as a conclusion rather than a hypothesis-generator) tends to premature closure.

---

**4) Immediate syllogistic conclusions (via Modus Ponens)**

$S(I)$   
**Annotation:** From  $\forall x (E(x) \rightarrow S(x))$  and  $E(I)$ , it follows that induction  $I$  is superior.

$L(A)$   
**Annotation:** From  $\forall x (P(x) \rightarrow L(x))$  and  $P(A)$ , it follows that abduction  $A$  is limited (not superior).

---

**5) Domain-dependence of IBE (Norton's constraint)**

$M(d)$  :  $d$  is a mature scientific domain  
**Annotation:** Predicate  $M(d)$  indicates domain  $d$  has well-developed background theory and tested alternatives.

$R(x, d)$  : method  $x$  is reliable in domain  $d$   
**Annotation:** Predicate  $R(x, d)$  states that method  $x$  tracks truth in domain  $d$ .

$\forall d (M(d) \rightarrow R(A, d))$   
**Annotation:** Where background science is mature  $M(d)$ , abduction  $A$  can be reliable.

$\forall d (\neg M(d) \rightarrow P(A))$   
**Annotation:** Outside mature domains  $\neg M(d)$  (e.g., miracle claims), abduction  $A$  tends toward premature closure  $P(A)$ .

$d_{mir}$  : the domain of miracle claims  
**Annotation:** Constant  $d_{mir}$  denotes miracle-claim contexts.

$\neg M(d_{mir})$   
**Annotation:** Miracle domains lack maturity in the Norton sense; hence not  $M$ .

$P(A)$   
**Annotation:** Therefore, in  $d_{mir}$ , abduction  $A$  again has the property  $P$  (premature closure), reinforcing  $L(A)$ .

Figure C1

Extended Symbolic Logic Formulation (1/2)

**6) Bayesian backbone (priors and unconceived alternatives)**

$H$  : target hypothesis (e.g., resurrection)  
**Annotation:** Constant  $H$  is the theistic target hypothesis.  
 $E$  : evidence (e.g., testimony, empty-tomb narratives)  
**Annotation:** Constant  $E$  denotes the evidential corpus under discussion.  
 $\neg H$  : the disjunction of all rivals to  $H$   
**Annotation:**  $\neg H$  collects known and unknown alternatives to  $H$ .  

$$P(H | E) = \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|\neg H)P(\neg H)}$$
**Annotation:** Posterior credence in  $H$  is constrained by its prior  $P(H)$  and the likelihoods under  $H$  and  $\neg H$ .  
 $\neg H = R_1 \vee \dots \vee R_n \vee U$   
**Annotation:** Decompose  $\neg H$  into known rivals  $R_i$  and a residual unknown class  $U$  (unconceived alternatives).  

$$P(H | E) = \frac{P(E|H)P(H)}{P(E|H)P(H) + \sum_i P(E|R_i)P(R_i) + P(E|U)P(U)}$$
**Annotation:** Any nonzero  $P(U)$  diverts probability mass away from  $H$ , lowering  $P(H | E)$ .  
 If  $P(H)$  is tiny and  $P(E | \neg H)$  is not negligible, then  $P(H | E)$  remains tiny.  
**Annotation:** Low base rates for  $H$  combined with plausible alternatives keep the posterior small (illustrated numerically in the scenarios).

**7) Worked posterior illustrations (miracle domain)**

$P(H) = 10^{-6}$ ,  $P(E | H) = 0.95$ ,  $P(E | \neg H) = 0.02 \Rightarrow P(H | E) \approx 4.8 \times 10^{-5}$   
**Annotation:** Even very favorable likelihoods with a tiny prior leave  $P(H | E)$  near zero.  
 $P(H) = 0.001$ ,  $P(E | H) = 0.7$ ,  $P(E | \neg H) = 0.05 \Rightarrow P(H | E) \approx 0.014$   
**Annotation:** A theist-friendly prior still yields a low posterior once dependence and unknowns are admitted.

**8) Zeus analogy as structural caution**

When unknowns  $U$  are ignored,  $P(H | E)$  appears large; when  $P(U) > 0$ , it collapses.  
**Annotation:** The Zeus cases show apparent abductive strength evaporating once new rivals enter hypothesis space.

**9) Consolidated conclusions**

$S(I) \wedge L(A)$   
**Annotation:** Induction  $I$  is superior, abduction  $A$  (as a conclusion-driver in immature domains) is limited.  
 $\neg M(d_{\text{mir}}) \wedge P(A) \Rightarrow L(A)$  in  $d_{\text{mir}}$   
**Annotation:** In miracle contexts  $d_{\text{mir}}$ , abduction  $A$  retains  $P$ , hence  $L(A)$ .  
 $P(U) > 0 \Rightarrow P(H | E)$  stays small for extraordinary  $H$   
**Annotation:** Unconceived alternatives  $U$  systematically cap the posterior for extraordinary  $H$ .  
 $\therefore$  IBE cannot by itself justify high credence in  $H$  in  $d_{\text{mir}}$   
**Annotation:** Abduction alone does not support high credence in targets like a resurrection; the method must be supplemented by domain maturity and strong priors, which are absent here.

**Master Proof – Natural Deduction Layout**

$\forall x(E(x) \rightarrow S(x))$   
**Annotation:** If a reasoning method involves evidence testing and adaptability, then it is superior for rational decision-making.  
 $E(I)$   
**Annotation:** Induction involves evidence testing and adaptability.  
 $S(I)$   
**Annotation:** From the first two premises, induction is superior for rational decision-making.  
 $\forall x(P(x) \rightarrow L(x))$   
**Annotation:** If a reasoning method leads to premature conclusions without verification, then it is limited and not superior.  
 $P(A)$   
**Annotation:** Abduction (when treated as a conclusion-driver) leads to premature conclusions without verification.  
 $L(A)$   
**Annotation:** Therefore, abduction is limited and not superior for rational decision-making.  
 $M(d) \rightarrow R(A, d)$   
**Annotation:** In mature scientific domains, abduction can be reliable.  
 $\neg M(d) \rightarrow P(A)$   
**Annotation:** Outside mature scientific domains, abduction tends toward premature closure.  
 $\neg M(d_{\text{mir}})$   
**Annotation:** The domain of miracle claims is not mature.  
 $P(A)$   
**Annotation:** Therefore, in miracle contexts, abduction again leads to premature closure.  
 $P(A) \rightarrow L(A)$   
**Annotation:** Premature closure entails limitation.  
 $L(A)$   
**Annotation:** Hence, abduction is limited when applied to miracle claims.  

$$P(H | E) = \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|\neg H)P(\neg H)}$$
**Annotation:** Posterior probability of the target hypothesis depends on priors and likelihoods.  
 $\neg H = R_1 \vee \dots \vee R_n \vee U$   
**Annotation:** The negation of the hypothesis includes known rivals and unconceived alternatives.  

$$P(H | E) = \frac{P(E|H)P(H)}{P(E|H)P(H) + \sum_i P(E|R_i)P(R_i) + P(E|U)P(U)}$$
**Annotation:** Accounting for unknown alternatives always reduces the posterior of the target hypothesis.  
 $P(H) \ll 1 \wedge P(U) > 0 \Rightarrow P(H | E) \approx 0$   
**Annotation:** With extremely low priors and nonzero probability on unknowns, posterior probability remains negligible.  
 $\therefore S(I) \wedge L(A) \wedge (P(H | E) \approx 0)$   
**Annotation:** Therefore, induction is superior, abduction is limited in miracle domains, and extraordinary hypotheses like the resurrection retain negligible probability even after evidence.

Figure C2