The Incoherence of Eternal Damnation:

Proportional Justice, Emotional Stability, and the Failure of Three Justifications

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Thesis. Eternal damnation, as traditionally construed, cannot coherently serve the aims of punishment (rehabilitation, retribution, or appeasement of wrath) without violating proportional justice or presupposing emotional instability; when paired with doctrines like an indeterminate "age of accountability," it further yields developmental incoherence and predictable psychological harm. Building on formal criteria for legitimate punishment, this paper argues: (a) rehabilitation requires finitude and a horizon for demonstrated change; (b) retribution demands proportionality to finite offenses; and (c) appeasement by an emotionally stable agent must be satiable within finite bounds. We then evaluate prominent theological replies (infinite offense, transcendent justice, free-will self-exclusion, warning pedagogy, and the eternity of the soul) and show that each either undercuts intelligible justice or amplifies disproportionality. Finally, we analyze the belief-dynamic role of fear/hope appeals and the doctrine of "age of accountability," linking these to measurable well-being and distress effects. The net result is a comparative case against eternal damnation as a defensible component of a just, wise, and emotionally stable theism.

Introduction

The doctrine of eternal damnation is commonly defended by appeal to standard penological aims—rehabilitation, retribution, or the appeasement of justified wrath. A

justifiable punishment, however, must satisfy intelligible criteria: it must be finitely bounded if it is to rehabilitate; it must be proportionate if retributive; and it must be satiable if it serves the appeasement of a rational agent's indignation. This paper contends that eternal punishment defeats all three aims simultaneously. It also examines how adjunct teachings—notably an indeterminate "age of accountability" for children—generate developmental arbitrariness and predictable distress, and how fear/hope incentives can bypass evidence-sensitive belief formation. The argument advances a cumulative case: even granting favorable theological frameworks, eternal damnation fails the tests of proportional justice and emotional stability (Kvanvig, 1993; Walls, 1992; Talbott, 2013).

Punishment Aims: Minimal Adequacy Conditions

We adopt three modest adequacy conditions often presupposed in ordinary moral reasoning and legal practice:

a) **Rehabilitation:** Punishment is instrumentally justified if it constructively alters dispositions, with an observable post-punishment horizon for change.

b) **Retribution:** Punishment is intrinsically justified if it is proportionate to culpable wrongs committed by finite agents within finite histories.

c) **Appeasement:** Punishment may serve to settle justified indignation, but only if an emotionally stable, rational agent can be satisfied by a finite penalty commensurate with the wrong.

These conditions are intentionally minimal: abandoning them empties "punishment" of the reasons that render it legitimate.

Why Eternal Punishment Cannot Rehabilitate

Rehabilitation presupposes a terminus: the agent must have a future in which changed character can be exercised and verified. By definition, *eternal* punishment eliminates any post-penalty horizon. Therefore, as an instrument of reform, it is

Table 1

Aims of Punishment, Adequacy Conditions, and Why Eternal Damnation Fails

Aim	Adequacy Condition	Failure Under Eternal Damnation
Rehabilitation	Finite horizon for change; credible post-penalty verification of reform	No post-penalty horizon exists; hope and incentives for growth are structurally extinguished
Retribution	Proportionality to finite offenses by finite agents	Infinite penalty overshoots finite wrongs; status of offended party displaces act-based desert
Appeasement	Satiability of justified indignation by a finite fitting penalty	Insatiable wrath implied or appeasement abandoned; imputes instability or collapses to vengeance

self-defeating: it imposes unending incapacitation that precludes the very good it is supposed to produce. Even Anselmian or retributive defenses that valorize punishment as intrinsically good implicitly concede that rehabilitation is not the operative aim of an endless penalty (Haratine, 2024).

Motivational preconditions. If hope is constitutive of reform, a regime that extinguishes possible release removes the motivational scaffolding for transformation. By contrast, finite, staged penalties can coordinate with restorative practices and verifiable progress—goods unavailable under eternity.

Why Eternal Punishment Violates Proportional Retribution

Retributivism anchors desert in the gravity of wrongdoing by finite agents. Whatever one's theistic metaphysics, *human* actions and the sufferings they impose are temporally and causally finite. An *infinite* penalty therefore overshoots the offense by an

unbounded margin. Attempts to repair proportionality by appealing to the status of the offended party (e.g., God's "infinite worth") shift desert from the wrongdoer's finite act to the victim's rank, collapsing proportionality into status-measured vengeance and abandoning the idea that desert scales with wrongdoing (Kvanvig, 1993; Talbott, 2013).

Specificity test. If two agents commit identical finite offenses, but one is said to offend a higher-status victim, does desert multiply without limit? If so, proportionality is no longer indexed to what the offender did, but to who was offended—a criterion that undermines retributivism's internal rationale.

Why Appeasement by an Emotionally Stable Agent Must Be Finite

If punishment is said to appease divine wrath, then either (i) wrath is rationally bounded and thus satiable by a finite, fitting penalty, or (ii) wrath is insatiable and requires endless suffering. The second horn imputes emotional instability to the offended agent; the first horn denies the need for an *eternal* penalty. Choice-models that reconceive hell as self-exclusion do not solve proportionality; they reframe the harm as self-inflicted while tacitly presupposing unending, unrectified disorder (Walls, 1992; Talbott, 2013). Either way, appeasement does not justify eternity.

Comparative governance. In credible human governance, appeasement is constrained by fairness and ends when the fitting penalty is complete. Elevating appeasement beyond satiability degrades it into a mask for vengeance.

The "Age of Accountability": Developmental and Normative Incoherence

Within popular theology, the "age of accountability" attempts to shield children from eternal jeopardy until they reach an undefined threshold of awareness. Yet no non-arbitrary criterion is available: proposed thresholds vary widely, and development is individual-specific. The doctrine thereby pairs *indeterminacy* (no stable marker) with *catastrophic stakes* (eternal peril), producing a standard that is neither practicable nor fair

(Duncan, 2020; Wellum, 2018).

Developmental mismatch. Because cognitive control, future-consequence modeling, and abstract-religious reasoning mature on different timelines, any single gate for "accountability" risks penalizing children for capacities they do not yet possess.

Psychological Correlates of Hell-Centric Pedagogy

Empirical work indicates that punitive conceptions of the divine and hell beliefs correlate with measurable differences in well-being and social outcomes. Cross-national data find that belief in hell predicts lower crime, whereas belief in heaven predicts higher crime—a complex pattern not straightforwardly supportive of individual flourishing (Shariff & Rhemtulla, 2012). Importantly for pastoral ethics, experimental and correlational evidence connects hell beliefs with *negative* well-being effects, including anxiety and distress (Shariff & Aknin, 2014). Broader research on religious struggle—doubts, perceptions of divine punishment or abandonment—links such struggles with heightened psychological distress (Hill, Gonzalez, Burdette, Ellison, & Taylor, 2023). While such studies do not determine doctrinal truth, they inform prudential judgments about pedagogical strategies that predictably amplify chronic dread, scrupulosity, and developmental burden.

Theological Replies Assessed with Secular Parallels

We consider five established replies and add secular analogues to foreground proportionality and intelligibility constraints:

(1) Infinite Offense

Claim. Offenses against an infinite God merit infinite punishment.

Reply. This shifts desert from *finite wrongdoing* to the *status* of the offended party, abandoning proportionality as ordinarily understood and yielding the implausible result that identical finite acts merit different deserts solely by victim rank (Kvanvig, 1993).

Secular parallel. Modern legal systems reject victim-rank multipliers that explode finite wrongs into limitless penalties; such multipliers are seen as incompatible with equal protection and proportionality norms.

(2) Transcendent Justice

Claim. Divine justice outruns human proportionality; apparent disproportions are inscrutable.

Reply. Invoking inscrutability undercuts the *intelligibility* of justice claims themselves. If "just" no longer tracks proportional or rational criteria, appeals to justice risk becoming unfalsifiable honorifics rather than reasons (Talbott, 2013).

Secular parallel. In civil and criminal adjudication, reasons must be public and reviewable; opaque rationales are defeaters, not justifiers, of severe sanctions.

(3) Free-Will Self-Exclusion

Claim. Hell honors a settled choice of separation.

Reply. Choices by finite agents under limited cognition and mixed evidence cannot obviously ground *irrevocable infinity*. If eternal harm persists, either God *wills* the persistence (reintroducing retribution/appeasement concerns) or cannot remedy it (raising tensions about sovereignty and love) (Walls, 1992; Talbott, 2013).

Secular parallel. Consent doctrine requires capacity, information, and reversibility for extreme outcomes; irreversible, lifelong harms based on compromised consent are presumptively unjust.

(4) Loving Warning

Claim. The doctrine functions pedagogically to deter grave harm.

Reply. A deterrent that is both *endless* and *irrevocable* forfeits rehabilitative goods and, on present evidence, imposes predictable well-being costs that undermine pastoral aims (Shariff & Aknin, 2014).

Secular parallel. Evidence-based prevention emphasizes finite, remedial sanctions paired with education; terror-based pedagogy correlates with avoidance, not understanding.

(5) Eternal Soul, Eternal Desert

Claim. Because the soul is eternal, desert must be eternal.

Reply. The metaphysical duration of a subject does not fix proportionate desert for *finite* acts. If persons can develop eternally, eternity underdetermines penalty and may favor remedial horizons (Talbott, 2013; Adams, 1999).

Secular parallel. Sentencing guidelines scale with offense gravity and offender history, not with how long the offender might live.

Belief Dynamics Under Fear and Hope

Let B_t be doxastic strength at time t, E_t net evidential input, and F_t, H_t the salience of fear/hope with weights w_E, w_F, w_H and learning rate $\eta > 0$:

$$B_{t+1} \;=\; B_t \;+\; \eta\big(w_E E_t + w_F F_t + w_H H_t\big). \tag{1}$$

Annotation: This shows how affective drivers can increase or maintain belief even with weak or negative evidential input when $w_F F_t + w_H H_t$ dominates $|w_E E_t|$. A corresponding logistic adoption probability:

$$\Pr(B{=}1 \mid E, H, F) \;=\; \frac{1}{1 + e^{-(w_E E + w_H H + w_F F)}}. \tag{2}$$

Annotation: Large fear/hope weights push $\Pr(B{=}1)$ *toward 1, illustrating how emotional salience can swamp evidence in uptake.* These simple forms do not adjudicate truth; they clarify incentive structures by which maximalist depictions of hell/heaven predictably sustain adherence.

Broader Implications: Justice, Love, and Emotional Stability

A doctrine of punishment for finite agents that (i) forecloses rehabilitation, (ii) violates proportionality, and (iii) presupposes insatiable wrath is difficult to square with

claims of perfect justice and steadfast love. One may instead infer either that the doctrine misdescribes the divine, that the traditional model of hell is mistaken, or that justice-talk is being used non-intensionally (as an honorific without rational criteria). Philosophical and theological proposals that mitigate or reject eternal conscious torment—e.g., remedial models, annihilationism, or universalist hope—are motivated, in part, by these coherence pressures (Talbott, 2013; Adams, 1999).

Conclusion

Eternal damnation cannot meet minimal adequacy conditions for punishment. It fails as rehabilitation because it removes any horizon for demonstrated change; it fails as retribution because it is disproportionate to finite offenses; and it fails as appeasement because it imputes emotional insatiability to the offended agent or collapses justice into status-based vengeance. These failures are not accidental but structural: each arises from the very feature that defines the doctrine—*endlessness*.

Integrated synthesis. When the same doctrine independently violates finitude, proportionality, and satiability, the probability that it is a coherent component of just governance is correspondingly reduced. Attempts to rescue the view by appeal to victim status, inscrutability, or self-exclusion trade one defect for another, eroding either proportional justice, rational intelligibility, or emotional stability.

Pedagogical implications. For educational settings—and especially with children—the doctrine's structural defects interact with predictable psychological costs linked to fear-based pedagogy (Shariff & Aknin, 2014; Hill et al., 2023). Instruction that substitutes terror for understanding discourages authentic value formation and fosters chronic anxiety, scrupulosity, and distrust.

Constructive alternatives. Frameworks that preserve (a) finite, remedial sanctions proportioned to finite wrongs; (b) demonstrable rehabilitation horizons; and (c) emotionally

stable appeasement bounded by fitting penalty, better satisfy the minimal standards of just governance. Among extant theological options, annihilationism and remedial-universalist trajectories align more closely with these constraints than eternal conscious torment (Talbott, 2013; Adams, 1999).

Burden of proof. Given the cumulative failures across all three aims and the pedagogical liabilities, the rational burden lies with defenders of eternity to supply a model that restores proportionality, intelligibility, and emotional stability without reintroducing equivalent defects elsewhere. Absent such restoration, eternal damnation remains epistemically and pastorally indefensible.

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Appendix A

Formal Logical Analyses (Expanded)

Rehabilitation Aim

Let $\text{Purpose}(x, \text{Rehab})$ mean the punishment of x aims at rehabilitation; $\text{FinitePun}(x)$ that x's punishment is finite; $\text{PossibleChange}(x)$ that x can demonstrate post-punishment reform; $\text{EternalPun}(x)$ that x is eternally punished.

$$\forall x \big(\text{Purpose}(x, \text{Rehab}) \rightarrow [\text{FinitePun}(x) \wedge \text{PossibleChange}(x)] \big). \qquad \text{(A1)}$$

$$\forall x \big(\text{EternalPun}(x) \rightarrow [\neg \text{FinitePun}(x) \wedge \neg \text{PossibleChange}(x)] \big). \qquad \text{(A2)}$$

$$\therefore \ \forall x \big(\text{EternalPun}(x) \rightarrow \neg \text{Purpose}(x, \text{Rehab}) \big). \qquad \text{(A3)}$$

Annotation: If rehabilitation requires finitude and the chance to demonstrate change, an eternal sentence allows neither; hence it cannot serve rehabilitation.

Retributive Proportionality

Let $\text{Purpose}(x, \text{Retrib})$ denote retributive aim; $\text{OffenseFin}(x)$ that x's offense is finite; $\text{PunInf}(x)$ that x's punishment is infinite; $\text{Prop}(x)$ proportionality satisfied; $\text{EternalPun}(x)$ eternal punishment.

$$\forall x \big(\text{Purpose}(x, \text{Retrib}) \rightarrow \text{Prop}(x) \big). \qquad \text{(A4)}$$

$$\forall x \ \text{OffenseFin}(x). \qquad \text{(A5)}$$

$$\forall x \big(\text{EternalPun}(x) \rightarrow \text{PunInf}(x) \big). \qquad \text{(A6)}$$

$$\forall x \big([\text{OffenseFin}(x) \wedge \text{PunInf}(x)] \rightarrow \neg \text{Prop}(x) \big). \qquad \text{(A7)}$$

$$\therefore \ \forall x \big(\text{EternalPun}(x) \rightarrow \neg \text{Purpose}(x, \text{Retrib}) \big). \qquad \text{(A8)}$$

Annotation: Infinite penalties for finite offenses violate proportionality; thus eternal punishment cannot be justified as retribution.

Appeasement of Wrath and Stability

Let Stable(y) mean y is emotionally stable; WrathAppeased(y) wrath satiated; PunDur(x) duration of x's punishment; Finite(t)/Infinite(t) classification; Purpose(x, Appease(y)) appeasement aim.

$$\forall y \Big(\text{Stable}(y) \rightarrow \forall x \, \exists t \, [\text{Finite}(t) \wedge \text{PunDur}(x){=}t \wedge \text{Purpose}(x, \text{Appease}(y)) \rightarrow \text{WrathAppeased}(y)] \Big).$$

$$\text{(A9)}$$

$$\forall x \big(\text{EternalPun}(x) \rightarrow \text{Infinite}(\text{PunDur}(x)) \big). \tag{A10}$$

$$\forall y \forall x \big(\text{Infinite}(\text{PunDur}(x)) \rightarrow \neg \exists t \, [\text{Finite}(t) \wedge \text{WrathAppeased}(y) \text{ by time } t] \big). \tag{A11}$$

$$\text{Stable}(y). \tag{A12}$$

$$\therefore \ \forall x \big(\text{EternalPun}(x) \rightarrow \neg \text{Purpose}(x, \text{Appease}(y)) \big). \tag{A13}$$

Annotation: If stability entails satiability by a finite, fitting penalty, then an infinite penalty either signals instability or abandons appeasement as a rationale.

Appendix B

Belief-Dynamic Model Under Affective Salience

$$B_{t+1} \;=\; B_t \;+\; \eta\big(w_E E_t + w_F F_t + w_H H_t\big). \tag{B1}$$

Annotation: Fear/hope terms can dominate evidence-weighted updates, maintaining adherence even when $E_t \leq 0$.

$$\Pr(B{=}1 \mid E, H, F) \;=\; \frac{1}{1 + e^{-(w_E E + w_H H + w_F F)}}. \tag{B2}$$

Annotation: With large $w_F F + w_H H$, adoption probability approaches 1 regardless of modest or negative evidential input.

Symbolic Logic Formalization

Rehabilitation Aim

$\forall x. (Purpose(x, Rehab) \rightarrow [FinitePun(x) \land PossibleChange(x)])$
Annotation: For any x, if the punishment of x has the purpose of rehabilitation, then x's punishment must be finite and allow the possibility of demonstrating change.

$\forall x. (EternalPun(x) \rightarrow [\neg FinitePun(x) \land \neg PossibleChange(x)])$
Annotation: For any x, if x is eternally punished, then x's punishment is not finite and x cannot possibly show change.

$\therefore \forall x. (EternalPun(x) \rightarrow \neg Purpose(x, Rehab))$
Annotation: Therefore, eternal punishment cannot serve the purpose of rehabilitation.

Retributive Proportionality

$\forall x. (Purpose(x, Retrib) \rightarrow Prop(x))$
Annotation: For any x, if the punishment aims at retribution, then proportionality must be satisfied.

$\forall x. OffenseFin(x)$
Annotation: For any x, the offense committed is finite.

$\forall x. (EternalPun(x) \rightarrow PunInf(x))$
Annotation: For any x, if x receives eternal punishment, then x's punishment is infinite.

$\forall x. ([OffenseFin(x) \land PunInf(x)] \rightarrow \neg Prop(x))$
Annotation: For any x, if the offense is finite but the punishment is infinite, proportionality is violated.

$\therefore \forall x. (EternalPun(x) \rightarrow \neg Purpose(x, Retrib))$
Annotation: Therefore, eternal punishment cannot serve the purpose of retribution.

Appeasement of Wrath and Stability

$\forall y. (Stable(y) \rightarrow \forall x. \exists t. [Finite(t) \land PunDur(x) = t \land Purpose(x, Appease(y)) \rightarrow WrathAppeased(y)])$
Annotation: For any agent y, if y is emotionally stable, then for any x, there exists some finite time t such that punishment of duration t fulfills the purpose of appeasement and y's wrath is satisfied.

$\forall x. (EternalPun(x) \rightarrow Infinite(PunDur(x)))$
Annotation: For any x, eternal punishment implies that the duration of x's punishment is infinite.

$\forall y \forall x. (Infinite(PunDur(x)) \rightarrow \neg \exists t. [Finite(t) \land WrathAppeased(y), by, time, t])$
Annotation: For any y and x, if the punishment duration is infinite, then there does not exist a finite time at which y's wrath is appeased.

$Stable(y)$
Annotation: Assume the offended party y is emotionally stable.

$\therefore \forall x. (EternalPun(x) \rightarrow \neg Purpose(x, Appease(y)))$
Annotation: Therefore, eternal punishment cannot serve the purpose of appeasement if the agent is emotionally stable.

A Fitch-Style Proof.

Fitch 1 — Rehabilitation Aim

1. $\forall x (Purpose(x, Rehab) \rightarrow (FinitePun(x) \land PossibleChange(x)))$.   Premise
Annotation: For any x, if the punishment aims at rehabilitation, then it must be finite and allow the offender a chance to change.

2. $\forall x (EternalPun(x) \rightarrow (\neg FinitePun(x) \land \neg PossibleChange(x)))$.   Premise
Annotation: For any x, if the punishment is eternal, then it is not finite and change is not possible.

3. $EternalPun(a)$.   Assumption
Annotation: Assume, for an arbitrary a, that eternal punishment applies.

4. $EternalPun(a) \rightarrow (\neg FinitePun(a) \land \neg PossibleChange(a))$. $\forall$-elim on (2)
Annotation: Specialize line (2) to the case of a.

5. $\neg FinitePun(a) \land \neg PossibleChange(a)$.   MP from (3),(4)
Annotation: From (3) and (4), it follows that a has neither finite punishment nor possible change.

6. $\neg PossibleChange(a)$. $\land$-elim on (5)
Annotation: Extract the second conjunct from (5).

7. $Purpose(a, Rehab) \rightarrow (FinitePun(a) \land PossibleChange(a))$. $\forall$-elim on (1)
Annotation: Specialize line (1) to a.

8. $Purpose(a, Rehab)$.   Assumption
Annotation: Suppose rehabilitation is the purpose of punishing a.

9. $FinitePun(a) \land PossibleChange(a)$.   MP from (7),(8)
Annotation: If rehabilitation is the purpose, both finitude and change must be present.

10. $PossibleChange(a)$. $\land$-elim on (9)
Annotation: From (9), extract the possibility of change for a.

11. $\bot$.   from (6),(10)
Annotation: Contradiction: $PossibleChange(a)$ and $\neg PossibleChange(a)$ both appear.

12. $\neg Purpose(a, Rehab)$.   neg-intro from (8)-(11)
Annotation: Therefore, rehabilitation cannot be the purpose of eternal punishment.

13. $EternalPun(a) \rightarrow \neg Purpose(a, Rehab)$.   cond-intro from (3)-(12)
Annotation: Discharge assumption (3) to derive the conditional.

14. $\forall x (EternalPun(x) \rightarrow \neg Purpose(x, Rehab))$. $\forall$-intro from (13)
Annotation: Generalize to all x: eternal punishment cannot serve rehabilitation.

Fitch 2 — Retributive Proportionality

15. $\forall x (Purpose(x, Retrib) \rightarrow Prop(x))$.   Premise
Annotation: For any x, if punishment aims at retribution, proportionality must hold.

16. $\forall x OffenseFin(x)$.   Premise
Annotation: All offenses are finite in scope.

17. $\forall x (EternalPun(x) \rightarrow PunInf(x))$.   Premise
Annotation: For any x, eternal punishment entails an infinite penalty.

18. $\forall x ((OffenseFin(x) \land PunInf(x)) \rightarrow \neg Prop(x))$.   Premise
Annotation: For any x, a finite offense combined with an infinite punishment breaks proportionality.

19. $EternalPun(a)$.   Assumption
Annotation: Assume eternal punishment for arbitrary a.

20. $EternalPun(a) \rightarrow PunInf(a)$. $\forall$-elim on (17)
Annotation: Specialize (17) to a.

21. $PunInf(a)$.   MP from (19),(20)
Annotation: From eternal punishment for a, its penalty is infinite.

22. $OffenseFin(a)$. $\forall$-elim on (16)
Annotation: From (16), a's offense is finite.

23. $(OffenseFin(a) \land PunInf(a)) \rightarrow \neg Prop(a)$. $\forall$-elim on (18)
Annotation: Specialize (18) to a.

24. $OffenseFin(a) \land PunInf(a)$. $\land$-intro from (21),(22)
Annotation: Combine (21) and (22).

25. $\neg Prop(a)$.   MP from (23),(24)
Annotation: Proportionality fails for a.

26. $Purpose(a, Retrib) \rightarrow Prop(a)$. $\forall$-elim on (15)
Annotation: If retribution is the purpose for a, proportionality must hold.

27. $Purpose(a, Retrib)$.   Assumption
Annotation: Suppose retribution is the purpose of punishing a.

28. $Prop(a)$.   MP from (26),(27)
Annotation: From (26) and (27), proportionality must hold.

29. $\bot$.   from (25),(28)
Annotation: Contradiction: $Prop(a)$ and $\neg Prop(a)$.

30. $\neg Purpose(a, Retrib)$.   neg-intro from (27)-(29)
Annotation: Therefore, retribution cannot be the purpose of eternal punishment.

31. $EternalPun(a) \rightarrow \neg Purpose(a, Retrib)$.   cond-intro from (19)-(30)
Annotation: Discharge assumption (19) to form the conditional.

32. $\forall x (EternalPun(x) \rightarrow \neg Purpose(x, Retrib))$. $\forall$-intro from (31)
Annotation: Generalize to all x: eternal punishment cannot serve retribution.

Fitch 3 — Appeasement and Emotional Stability

33. $Stable(y)$.   Assumption
Annotation: Assume an arbitrary offended agent y is emotionally stable.

34. $Stable(y) \rightarrow \forall x \exists t (Finite(t) \land (Purpose(x, Appease(y)) \rightarrow AppeasedBy(y, t)))$.   Premise
Annotation: If y is stable, then for each x there is a finite time that would suffice for appeasement if appeasement is the aim.

35. $\forall x (EternalPun(x) \rightarrow Infinite(PunDur(x)))$.   Premise
Annotation: Eternal punishment implies infinite duration.

36. $\forall x (Infinite(PunDur(x)) \rightarrow \neg \exists t (Finite(t) \land AppeasedBy(y, t)))$.   Premise
Annotation: Infinite duration prevents any finite time from appeasing y.

37. $EternalPun(a)$.   Assumption
Annotation: Assume eternal punishment for arbitrary a.

38. $Infinite(PunDur(a))$.   from (35),(37)
Annotation: From eternal punishment, the duration for a is infinite.

39. $\neg \exists t (Finite(t) \land AppeasedBy(y, t))$.   from (36),(38)
Annotation: From infinite duration, there can be no finite appeasement time.

40. $\forall x \exists t (Finite(t) \land (Purpose(x, Appease(y)) \rightarrow AppeasedBy(y, t)))$.   from (33),(34)
Annotation: From stability, there exists a finite time for each x if appeasement is the aim.

41. $\exists t (Finite(t) \land (Purpose(a, Appease(y)) \rightarrow AppeasedBy(y, t)))$. $\forall$-elim on (40)
Annotation: Specialize (40) to a.

42. $Finite(c) \land (Purpose(a, Appease(y)) \rightarrow AppeasedBy(y, c))$. $\exists$-elim on (41)
Annotation: Choose a fresh witness c for the finite appeasement time.

43. $Purpose(a, Appease(y))$.   Assumption
Annotation: Suppose appeasement is the purpose of punishing a.

44. $Purpose(a, Appease(y)) \rightarrow AppeasedBy(y, c)$. $\land$-elim on (42)
Annotation: From (42), if appeasement is the purpose, then y is appeased at c.

45. $AppeasedBy(y, c)$.   MP from (43),(44)
Annotation: From assumption, y is appeased at c.

46. $Finite(c)$. $\land$-elim on (42)
Annotation: The chosen witness c is finite.

47. $\exists t (Finite(t) \land AppeasedBy(y, t))$. $\exists$-intro from (45),(46)
Annotation: Therefore, there exists a finite time at which y is appeased.

48. $\bot$.   from (39),(47)
Annotation: Contradiction: both existence and non-existence of finite appeasement.

49. $\neg Purpose(a, Appease(y))$.   neg-intro from (43)-(48)
Annotation: Therefore, appeasement is not the purpose of eternal punishment.

50. $EternalPun(a) \rightarrow \neg Purpose(a, Appease(y))$.   cond-intro from (37)-(49)
Annotation: Discharge assumption (37) to form the conditional.

51. $\forall x (EternalPun(x) \rightarrow \neg Purpose(x, Appease(y)))$. $\forall$-intro from (50)
Annotation: Generalize to all x: eternal punishment cannot serve appeasement.

Consolidated Master Result

52. $\forall x (EternalPun(x) \rightarrow (\neg Purpose(x, Rehab) \land \neg Purpose(x, Retrib) \land \neg Purpose(x, Appease(y))))$.   from (14),(32),(51)
Annotation: Consolidated result: for any x, eternal punishment fails rehabilitation, retribution, and appeasement.

Figure B1

Extended Symbolic Logic Formulation.