

The Valuation Problem in Justice Theory: Incommensurability and the Categorical Failure of PSA

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This paper identifies a fundamental category error in penal substitutionary atonement theory that has been overlooked in existing critiques. While previous objections have focused on mathematical proportionality (finite versus infinite duration), I argue that the deeper problem lies in the non-fungibility of duration-based penalties. Drawing on theories of incommensurability in justice and value theory, I demonstrate that certain penalties—particularly those defined by temporal duration—cannot be satisfied through value-equivalent substitution, regardless of the substitute’s worth or the affection held for them by the judging authority. This incommensurability argument reveals that penal substitutionary atonement commits what I term the *Valuation Fallacy*: the mistaken belief that high-value substitutes can satisfy categorically distinct penalty types. This critique is stronger than mere proportionality objections because it shows that even if one grants infinite value to Christ’s sacrifice, duration-based penalties remain categorically unsatisfiable through non-durational means. The implications extend beyond theology to general theories of punishment, substitutionary justice, and the limits of commensuration in legal systems.

Introduction

The doctrine of penal substitutionary atonement claims that Jesus Christ’s death satisfies the penalty humanity owes for sin. On a common formulation, sin merits eternal

punishment—traditionally, eternal conscious torment—yet God’s justice is satisfied when Christ bears that penalty in our stead. The substitution is judged valid even though Christ’s suffering is temporally finite (roughly three days), whereas the penalty it allegedly satisfies is eternal.

Proportionality objections question how a finite-duration event equals an infinite-duration penalty. Standard replies appeal to the infinite worth of Christ or the inscrutability of divine justice. Both responses presume commensurability: that the relevant properties can be placed on a shared metric so that “more” of one compensates for “less” of another. I argue that this presupposition is false. The deeper problem is *dimensional mismatch*: duration-based penalties are non-fungible with value-based payments. Attempting to satisfy a duration requirement with a value payment enacts a category error—the *Valuation Fallacy*.

The paper proceeds by formalizing fungibility and incommensurability (Chang, 1997, 2002; Raz, 1986), classifying penalty types with attention to those defined constitutively by time, identity, and qualitative constraints (Duff, 2001; Zehr, 2015), arguing that penal substitution trades in an illicit cross-dimensional substitution, and answering leading rejoinders from the literature and its critics (Boersma, 2004; Finlan, 2005; Girard, 1986; Green & Baker, 2000; Hart, 2019; Jeffery et al., 2007; Stump, 2018; Wright, 2016).

Theoretical Framework: Fungibility and Incommensurability in Justice

Fungibility in penalties

Definition 1 (Fungible Penalty). A penalty P is *fungible* if and only if there exists a dimension of value V such that any disposition achieving $V(P) = v$ suffices to meet the justice requirement for the offense.

Monetary fines are paradigmatically fungible: any \$100 settles a \$100 fine, independent of payer identity, provided the value condition is met.

Non-fungible penalties

Definition 2 (Non-Fungible Penalty). A penalty P is *non-fungible* if and only if it is individuated by properties that cannot be satisfied via value-equivalent substitution on any single commensurating scale.

Imprisonment illustrates non-fungibility: a twenty-year sentence cannot be justly replaced by payment, even at the cost-equivalent of incarceration. The penalty is individuated by (i) personal identity of the offender, (ii) time served, and (iii) a qualitative mode of deprivation (of liberty, not wealth) (Duff, 2001).

Incommensurability

In value theory, items are incommensurable when they lack a common unit of measure suitable for trade-off calculations, even when they remain comparable in other senses (Chang, 1997, 2002; Raz, 1986). This matters for sanctions because some penalty properties (for example, time) do not admit compensatory exchange against others (for example, money) without altering the nature of what the penalty is.

Lemma 1 (Dimensional Incommensurability). If penalty P_1 is defined by property D_1 and penalty P_2 by D_2 , and D_1, D_2 are dimensionally incommensurable, then no amount of P_2 satisfies P_1 . *Notes: If time makes a punishment what it is, you cannot pay value instead and still have the same punishment.*

Commensuration and cross-dimensional failure

Definition 3 (Commensuration Function). Let $(\mathcal{T}, +)$ be temporal durations and $(\mathcal{V}, +)$ value magnitudes. A commensuration function is a mapping $F : \mathcal{T} \rightarrow \mathcal{V}$ such that for any penalty defined essentially by duration $t \in \mathcal{T}$ and identity i , satisfaction requires t served by i . F commensurates if and only if (i) it preserves the operation ($F(t_1 + t_2) = F(t_1) + F(t_2)$) and (ii) preserves satisfaction conditions (if t is required, then $F(t)$ suffices whenever delivered by any suitable bearer).

Lemma 2 (No Homomorphism with Satisfaction Preservation). No mapping $F : \mathcal{T} \rightarrow \mathcal{V}$ can be both an additive homomorphism and preserve satisfaction for duration-defined penalties with identity-constraints. Hence, no amount of value discharges an essentially durational sanction. *Notes: The failure is by kind-mismatch, not by magnitude; homomorphisms on magnitudes ignore constitutive identity and the mode of deprivation.* See Raz (1986) and Chang (1997).

Penalty Types and the Duration Category

Judge analogy (intuitive anchor)

Notes: Imagine a judge sentences one million offenders to life imprisonment. The judge's beloved son volunteers to serve three days, claiming his "infinite value" makes those three days equal the million lifetimes. Our intuition balks: the penalty is not defined by value but by time served. This sets up the taxonomy that follows.

A working taxonomy

Type 1: Value-Based. Fines, restitution, forfeitures (fungible along a value scale).

Type 2: Duration-Based. Imprisonment, probationary periods, suspensions, mandated service hours (constitutively temporal).

Type 3: Status-Based. Civil disabilities, registry duties, loss of civic standing (identity or status constitutive).

Type 4: Corporal. Physical interventions such as lashes, execution, or bodily harms (qualitative act-type constitutive).

Why duration-defined penalties resist substitution

Three jointly sufficient features render duration penalties non-fungible.

First, there is a personal identity constraint: the deprivations are indexed to the offender. Serving time is conceptually tied to that person's biography; transferring it alters the act-type (Duff, 2001).

Second, there is temporal constitutivity: duration is not an incidental measure of severity but part of the penalty's essence; "time served" names a constitutive satisfaction condition.

Third, there is a qualitative mode: the deprivation is of liberty-through-time, not generic disutility. Replacing time with money or third-party pain changes the penalty-kind.

Identity-locked durational sanctions

Lemma 3 (Identity Lock). If a sanction's essence includes (a) deprivation of liberty over interval d and (b) biographical indexing to offender i , then for any $j \neq i$, $\text{Serves}(j, d)$ is not identical in kind to $\text{Serves}(i, d)$; it is a distinct institutional act. Thus, substitution is conceptually limited to in-kind and in-duration, not cross-currency value delivery.

Notes: Even if a parent could legally volunteer to serve a child's term, the only way to satisfy a twenty-year sentence is that *someone serves twenty years*. Paying money or intensifying pain swaps the penalty-kind and fails satisfaction (Duff, 2001).

The Substitution Impossibility Theorem

Theorem 1 (Substitution Impossibility for Duration Penalties). Let P_d be a duration-based penalty defined by duration d and personal identity i . Let V be any value-based payment. Then V cannot satisfy P_d for any finite or infinite magnitude of V , because:

1. P_d requires that individual i experience state S for duration d ;
2. V provides value v but not duration d served by i ;
3. Duration and value are dimensionally incommensurable;
4. Therefore, V and P_d satisfy different evaluative criteria;
5. Therefore, $V \not\equiv P_d$ (that is, V cannot substitute for P_d).

Satisfaction, Commutation, and Remission

Distinct modalities of resolving sanctions

Satisfaction meets constitutive conditions of the sanction (kind, identity, and duration). **Commutation** changes the sanction by prerogative, waiving part or all of the constitutive conditions. **Remission** forgoes execution altogether. If a substitutionary theory is framed as satisfaction, it must meet durational essence when the sanction is durational. If it is framed as commutation, it concedes that justice is not satisfied but waived by mercy or authority, which is a different model (Duff, 2001; Murphy & Hampton, 1988; Zehr, 2015).

Why fines substitute and time does not

Fines function on a value dimension and so admit payer-substitution without altering the sanction-kind. By contrast, imprisonment communicates censure through time-indexed civic exclusion; substituting money or third-party pain distorts the communicative content and fails retributive fit (Duff, 2001). Restorative settlements operate on a repair dimension and so differ categorically from liberty-deprivation (Zehr, 2015).

Application to Penal Substitutionary Atonement

Characterizing the alleged penalty

Under eternal conscious torment models, the penalty is explicitly durational: “eternal punishment,” “forever and ever,” and similar formulations make temporal extension constitutive, not incidental (Hart, 2019; Stump, 2018; Wright, 2016). If the essence is *bad forever*, then the relevant satisfaction condition is durational, personal, and relationally indexed.

The Valuation Fallacy

Definition 4 (Valuation Fallacy). Illicitly treating a duration-defined penalty as fungible along a value dimension; that is, inferring from high (even infinite) value of a substitute that the substitute thereby satisfies a durational requirement.

Core argument (seven-premise layout)

- (1) Justice requires meeting a penalty's definitional conditions (its constitutive kind-properties), rather than merely producing compensatory value.
- (2) Eternal conscious torment is a penalty whose essence is durational with personal/relational indexing.
- (3) Christ's passion is finite in duration.
- (4) Value and duration are dimensionally incommensurable (Chang, 1997; Raz, 1986).
- (5) If value and duration are incommensurable, then appeal to value (even infinite) cannot substitute for an essentially durational requirement (by Lemma 1).
- (6) Therefore, appeal to Christ's value/status cannot convert a finite-duration act into satisfaction of an essentially durational penalty (by Theorem 1).
- (7) Hence, penal substitution—if construed as satisfying eternal conscious torment via Christ's finite passion by appeal to his value/status—fails by category error.

Conclusion: The substitution as typically framed does not meet the durational essence of the alleged penalty.

Anticipating and Answering Stronger Rejoinders**Objection 1: Infinite intensity or qualitative infinity**

Claim. The quality of Christ's suffering is infinite; intensity compensates for short duration.

Reply. Intensity and duration are distinct constitutive properties. Sentencing practice marks this: harsher conditions do not transform a one-year sentence into a twenty-year sentence. Altering quality does not satisfy a time requirement. The attempt

presupposes a common currency between intensity and time that value theory denies (Chang, 2002).

Objection 2: Metaphysical compression of eternity

Claim. An infinite being can compress eternity into three days.

Reply. Either compression alters the penalty-kind (no longer duration) or it posits that three days are eternity, which contradicts temporal semantics. If the penalty is redefined as “bearing the worth of eternity,” the view has tacitly abandoned a duration-defined penalty for a value-defined one, which concedes the critique.

Objection 3: Atemporality of the divine act

Claim. God is atemporal; Christ’s atoning act is timelessly present and thus coextensive with eternity.

Reply. Atemporal presence is not temporal extension. If the sanction’s essence is endless conscious duration, atemporal acts are categorically different kinds of satisfaction. Redefining the penalty as “timeless divine act” changes the constitutive property of the sanction; it is a category shift, not a solution.

Objection 4: Federal headship and union with Christ

Claim. In covenantal union, Christ’s acts are ours; thus his satisfaction is ours.

Reply. Imputation may transfer status, but it cannot retro-create time served by the offender. If penal substitution rests on headship, it tacitly changes the sanction-kind from durational to status-based, which is a different explanandum and requires independent defense.

Objection 5: Commutation analogies in law

Claim. Executives commute sentences; penalties can be remitted or exchanged.

Reply. Commutation is not satisfaction; it waives satisfaction by prerogative. If penal substitution is a commutation story, then justice is not satisfied by substitution but set

aside by mercy or fiat, which is a different model (Duff, 2001; Zehr, 2015).

Objection 6: Infinite offense and infinite penalty scaling

Claim. Offense against an infinite God warrants an infinite penalty; Christ, being infinite, balances the scale.

Reply. “Infinite” here equivocates between magnitude on a value scale and kind of property. Even if guilt magnitude is unbounded, it does not follow that a value payment discharges a time-constituted sanction. The scale of offense does not determine the currency of satisfaction. The trade presumes cross-dimensional exchange that incommensurability forbids (Raz, 1986).

Broader Implications

For atonement theory

The critique does not deny that Christian theology can explain reconciliation; it targets a specific transactional mechanism. Non-transactional models such as Christus Victor, participatory or theosis models, moral influence, and scapegoat demystification avoid the category error by refusing to cash reconciliation out in terms of penalty-payment (Aulén, 1931; Boersma, 2004; Girard, 1986; Green & Baker, 2000; Hart, 2019; Stump, 2018; Wright, 2016). Penal substitution can be salvaged only by either abandoning eternal conscious torment in favor of a non-durational penalty such as annihilation, or by re-describing the penalty-kind away from duration, which concedes the main point.

Retributive fit and the limits of mercy

On a Kantian picture, retribution demands kind-matching between offense and sanction, not utility-equating across currencies; altering kind forfeits retributive fit. Mercy such as remission or commutation can be coherent, but it is not satisfaction (Kant, 1996; Murphy & Hampton, 1988). A penal substitution account must choose: either genuine satisfaction, and so in-kind duration when the penalty is durational, or mercy, and so

non-satisfaction.

Comparative Atonement Models

Christus Victor

Christ's death and resurrection are a victory over sin, death, and demonic powers; reconciliation is achieved by liberation rather than debt-payment (Aulén, 1931).

Notes: This avoids cross-dimensional substitution entirely because it is not a payment model; satisfaction talk is replaced by deliverance.

Moral Influence

Christ's self-giving love transforms human hearts and moral agency, drawing humanity into reconciliation without a transactional settlement (Green & Baker, 2000).

Notes: Emphasizes moral psychology and relational change; again, not a payment model, so no need to convert value into duration.

Participatory / Theosis

Believers are united to Christ, participating in his death and resurrection; reconciliation is effected by ontological transformation rather than penalty transfer (Stump, 2018).

Notes: Shifts the explanans from law-court satisfaction to union and deification; category mismatch dissolves.

Recapitulation

Christ relives and perfects the human story, reversing Adam's disobedience and restoring humanity (McIntyre, 1992).

Notes: Focuses on restorative ontology, not payment; no cross-currency substitution is required.

Scapegoat / Girardian

The crucifixion exposes and breaks the mechanism of sacred violence and scapegoating, reconfiguring sacrifice (Girard, 1986).

Notes: A critique of sacrificial payment—atonement happens through unveiling and transformation of communal desire, not settlement.

Conclusion

If the penalty is essentially durational, then a non-durational value payment, however exalted, is the wrong currency. Appeals to infinite worth, intensity, timelessness, or headship each either trade on forbidden cross-dimensional exchange or revise the penalty's essence. That is the *Valuation Fallacy*. The path forward is conceptual clarity: either re-characterize the penalty-kind coherently, or adopt atonement models that do not require cross-dimensional substitution.

Formal Symbolic Logic

Language and domain

Domain of Discourse: \mathcal{P} (penalties), \mathcal{A} (agents), \mathcal{T} (durations including ∞), \mathcal{V} (values including ∞), \mathcal{D} (dimensions). *Notes:* *The domain lets us track which properties matter for what counts as “satisfying” a penalty.*

Predicates: $\text{Duration}(p, t)$, $\text{Value}(a, v)$, $\text{Serves}(a, p, t)$, $\text{Satisfies}(a, p)$, $\text{Incomm}(d_1, d_2)$, $\text{Just}(p)$, $\text{Fungible}(p)$. *Notes:* *“Duration” says a penalty essentially requires time; “Satisfies” is the target we are analyzing.*

Axioms

$$D1: \forall p, t [\text{Duration}(p, t) \rightarrow \forall a (\text{Satisfies}(a, p) \rightarrow \text{Serves}(a, p, t))] \quad (1)$$

$$D2: \text{Incomm}(\text{duration}, \text{value}) \quad (2)$$

$$D3: \forall d_1, d_2 [\text{Incomm}(d_1, d_2) \rightarrow \neg \exists f : d_1 \rightarrow d_2] \quad (3)$$

$$D4: \forall p, t [\text{Duration}(p, t) \rightarrow \neg \text{Fungible}(p)] \quad (4)$$

$$D5: \forall p [\text{Just}(p) \rightarrow \exists a \text{Satisfies}(a, p)] \quad (5)$$

Notes: D1 encodes that satisfying a time-defined penalty requires serving the time; D2–D3 encode that time and value cannot be converted; D4 that time-penalties are non-fungible; D5 that justice demands satisfaction by someone.

Two principles for cross-dimension blocks**P6 (No Cross-Currency Satisfaction).**

$\forall p \in \mathcal{P} (\text{Duration}(p, t) \wedge \text{Value}(a, v)) \Rightarrow \neg(\text{Satisfies}(a, p) \text{ by } v)$. *Notes:* Having (even infinite) value does not, by itself, count as time served.

P7 (Identity-Constitutivity). If p encodes offender identity i essentially, then for $j \neq i$, $\text{Serves}(j, p, t)$ cannot be identical in kind to $\text{Serves}(i, p, t)$; any satisfaction via j must still instantiate t fully, else it is a commutation, not satisfaction. *Notes:* Who serves matters to the kind of act the system performs.

Theorem (Value cannot satisfy duration)

$$\forall p, a, t, v [\text{Duration}(p, t) \wedge \text{Value}(a, v) \wedge t \neq \infty \rightarrow (\text{Serves}(a, p, t) \not\leftrightarrow \text{Satisfies}(a, p))]$$

Explanation. Having value does not entail serving the requisite time; hence value alone cannot satisfy a duration-defined penalty.

Penal substitution instance

Let j be Jesus and e the eternal conscious torment penalty. Given $\text{Duration}(e, \infty)$, $\text{Value}(j, \infty)$, $\text{Serves}(j, e, 3)$, and $3 \neq \infty$, then from D1, $\text{Satisfies}(j, e) \rightarrow \text{Serves}(j, e, \infty)$, contradicting $\text{Serves}(j, e, 3)$. Therefore $\neg\text{Satisfies}(j, e)$. *Notes: If eternity is required, three days is not enough, regardless of who serves it or how valuable they are.*

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Symbolic Logic Formulation

Language and Domain

Domain of Discourse: P, A, T, V, D

Annotation: The entities we reason about are **penalties** (P), **agents** (A), **durations** (T), **values** (V), and **dimensions** (D).

Predicates: $\text{Duration}(p, t)$, $\text{Value}(a, v)$, $\text{Serves}(a, p, t)$, $\text{Satisfies}(a, p)$, $\text{Incomm}(d_1, d_2)$, $\text{Just}(p)$, $\text{Fungible}(p)$

Annotation: These predicates describe relationships such as "penalty p requires duration t ," "agent a has value v ," "agent a serves penalty p for time t ," "agent a satisfies penalty p ," "two dimensions are incommensurable," and so on.

Axioms

D1: $\forall p, t [\text{Duration}(p, t) \rightarrow \forall a (\text{Satisfies}(a, p) \rightarrow \text{Serves}(a, p, t))]$
Annotation: If a penalty is defined by a time requirement, then anyone who satisfies it must serve that time.

D2: $\text{Incomm}(\text{duration}, \text{value})$
Annotation: Duration and value are incommensurable kinds – they can't be measured on a common scale.

D3: $\forall d_1, d_2 [\text{Incomm}(d_1, d_2) \rightarrow \neg \exists f : d_1 \rightarrow d_2]$
Annotation: If two dimensions are incommensurable, there exists no conversion function from one to the other – you can't trade time for value in any legitimate way.

D4: $\forall p, t [\text{Duration}(p, t) \rightarrow \neg \text{Fungible}(p)]$
Annotation: Any time-defined penalty is non-fungible; it can't be replaced or commuted by other forms of payment or value.

D5: $\forall p [\text{Just}(p) \rightarrow \exists a \text{Satisfies}(a, p)]$
Annotation: For justice to hold, each just penalty must be satisfied by someone.

Cross-Dimensional Block Principles

P6 (No Cross-Currency Satisfaction): $\forall p \in P [(\text{Duration}(p, t) \wedge \text{Value}(a, v)) \Rightarrow \neg (\text{Satisfies}(a, p) \text{ by } v)]$
Annotation: Having any degree of value, even infinite, does not by itself satisfy a time-based penalty. Value cannot function as temporal payment.

P7 (Identity-Constitutivity): $\forall i, j, p, t [(\text{EncodesIdentity}(p, t) \wedge j \neq i) \Rightarrow (\text{Serves}(j, p, t) \neq \text{Serves}(i, p, t))]$
Annotation: If the penalty is tied to a specific offender's identity, then having someone else serve it changes the nature of the act – it's no longer the same kind of fulfillment.

Theorem: Value Cannot Satisfy Duration

$\forall p, a, t, e [\text{Duration}(p, t) \wedge \text{Value}(a, e) \wedge t \neq \infty \rightarrow (\text{Serves}(a, p, t) \neq \text{Satisfies}(a, p))]$
Annotation: Even if an agent has great or infinite value, if the penalty requires time, then serving less than the required duration fails to satisfy it. Value cannot substitute for duration.

Substitution Impossibility Applied to PSA

Let
 $j = \text{Jesus}$, $e = \text{Eternal Punishment}$
 and assume
 $\text{Duration}(e, \infty)$, $\text{Value}(j, \infty)$, $\text{Serves}(j, e, 3)$, $3 \neq \infty$

Then from D1:
 $\text{Satisfies}(j, e) \rightarrow \text{Serves}(j, e, \infty)$
 But this contradicts $\text{Serves}(j, e, 3)$
 Therefore:
 $\neg \text{Satisfies}(j, e)$

Annotation:
 If the penalty's essence is eternal duration, and Christ served only three days, the conditions for satisfaction are not met – regardless of Christ's infinite worth or divinity. The substitution fails by categorical mismatch rather than by mere magnitude.

A Fitch-Style Proof

- $\forall p, \forall t [\text{Duration}(p, t) \rightarrow \forall a (\text{Satisfies}(a, p) \rightarrow \text{Serves}(a, p, t))]$ (D1)
Annotation: If a penalty is defined by a time requirement, then anyone who satisfies it must serve exactly that required time.
- $\text{Duration}(e, \infty)$ Premise
Annotation: The penalty e (eternal punishment) is defined by infinite duration.
- $\text{Value}(j, \infty)$ Premise
Annotation: The substitute agent j (Jesus) has infinite value.
- $\text{Serves}(j, e, 3)$ Premise
Annotation: The duration of Jesus's suffering is three days, a finite temporal span.
- $3 \neq \infty$ Axiom of Arithmetic
Annotation: Three days is not equivalent to eternity.
- $\forall d_1, d_2 [\text{Incomm}(d_1, d_2) \rightarrow \neg \exists f : d_1 \rightarrow d_2]$ (D3)
Annotation: If two properties are incommensurable (like time and value), there is no legitimate conversion function between them.
- $\text{Incomm}(\text{duration}, \text{value})$ (D2)
Annotation: Duration and value are incommensurable dimensions; they cannot be exchanged on a single scale.
- $\forall p, \forall t [\text{Duration}(p, t) \rightarrow \neg \text{Fungible}(p)]$ (D4)
Annotation: Any penalty whose essence is time served cannot be replaced by another currency of satisfaction.
- $\forall p, [\text{Just}(p) \rightarrow \exists a \text{Satisfies}(a, p)]$ (D5)
Annotation: Justice requires that each penalty is satisfied by some agent, not necessarily that substitution is allowed.
- $\forall p, [(\text{Duration}(p, t) \wedge \text{Value}(a, v)) \rightarrow \neg (\text{Satisfies}(a, p) \text{ by } v)]$ (P6)
Annotation: Even infinite value does not satisfy a time-defined penalty, since the currencies are non-convertible.
- $\forall i, j, p, t, [(\text{EncodesIdentity}(p, t) \wedge j \neq i) \rightarrow (\text{Serves}(j, p, t) \neq \text{Serves}(i, p, t))]$ (P7)
Annotation: When a penalty is tied to a specific person, it cannot be fulfilled by another; the act type changes.
- $\forall a, p, t_1, t_2, [(\text{Serves}(a, p, t_1) \wedge t_1 \neq t_2) \rightarrow \neg \text{Serves}(a, p, t_2)]$ (Exactness)
Annotation: Serving a penalty for a specific duration excludes having served it for any different duration.
- $\text{Duration}(e, \infty) \rightarrow \forall a (\text{Satisfies}(a, e) \rightarrow \text{Serves}(a, e, \infty))$
 From 1, Universal Instantiation
Annotation: If the penalty requires eternity, any agent who satisfies it must have served for eternity.
- Assume $\text{Satisfies}(j, e)$ (Assumption for Indirect Proof)
Annotation: Suppose, for argument's sake, that Jesus's act does satisfy the eternal penalty.
- $\text{Serves}(j, e, \infty)$ From 13 and 14, Modus Ponens
Annotation: From the assumption and the requirement, it follows that Jesus must have served eternally.
- $\text{Serves}(j, e, 3) \wedge 3 \neq \infty$ From 4 and 5, Conjunction Introduction
Annotation: We note both the finite service and the difference between three days and eternity.
- $\neg \text{Serves}(j, e, \infty)$ From 12, Universal Instantiation
Annotation: By the rule of exact service, if Jesus served three days, he did not serve infinity.
- $\neg \text{Serves}(j, e, \infty)$ From 16 and 17, Modus Ponens
Annotation: Therefore, Jesus did not serve eternity.
- $\text{Serves}(j, e, \infty) \wedge \neg \text{Serves}(j, e, \infty)$ From 15 and 18, Conjunction Introduction
Annotation: A contradiction arises – Jesus both did and did not serve eternity.
- $\neg \text{Satisfies}(j, e)$ From 14–19, Indirect Proof
Annotation: Discharging the assumption, we conclude that Jesus does not satisfy the eternal penalty.
- $\forall a, [(\text{Duration}(e, \infty) \wedge \text{Serves}(a, e, 3)) \rightarrow \neg \text{Satisfies}(a, e)]$ Generalization
Annotation: For any agent, serving a finite duration cannot satisfy an eternal penalty.
- $\forall p, a, t, t', [(\text{Duration}(p, t) \wedge t' \neq t \wedge \text{Serves}(a, p, t')) \rightarrow \neg \text{Satisfies}(a, p)]$
 Schema from 1 and 12
Annotation: Generally, if the required duration differs from the one actually served, satisfaction fails – regardless of value.
- $\therefore \neg \text{Satisfies}(j, e)$ Final Result
Annotation: The penal substitution claim fails logically; a finite event cannot satisfy an essentially eternal, durational penalty. The incommensurability of time and value makes substitution impossible.

Figure 1

Extended Symbolic Logic Formulation.