

- Atonement theology loses its metaphysical foundation (if Christ isn't God-man, his death lacks claimed significance)
- Systematic theology's internal consistency collapses (since Incarnation connects Trinity, divine nature, human nature, soteriology, and eschatology)

This doesn't demonstrate Christianity is *false* (perhaps God exists but didn't become incarnate), but it demonstrates that Chalcedonian Christianity—the position of Catholic, Orthodox, and most Protestant traditions—rests on logical incoherence.

Epistemic Implication: Recognition of this contradiction doesn't require exotic knowledge or specialized training. The modal problem is accessible to anyone who understands the difference between "can" and "cannot." That such a fundamental incoherence persists at the center of Christian theology for nearly two millennia suggests that institutional momentum, social pressure, and motivated reasoning can sustain even obviously contradictory beliefs when the costs of recognition are sufficiently high.

Which returns us to Cognitocornus Dissonantia: the horns grow not from complexity but from the gap between what we recognize and what we admit.

II. Penal Substitutionary Atonement: Incommensurability

Problem Statement

Penal Substitutionary Atonement claims Christ's finite suffering (approximately three days) satisfies humanity's infinite penalty (eternal punishment). This requires equating finite duration with infinite duration—a mathematical impossibility.

Formal Demonstration: The Basic Incommensurability

PSA Incommensurability.

Let P = penalty for sin, C = Christ's payment, t = temporal duration, v = value/worth

Premises:

$$t_P = \lim_{n \rightarrow \infty} n = \infty \quad (\text{Penalty requires infinite temporal duration})$$

$$t_C = 3 \text{ days} = k \text{ where } k \in \mathbb{R}^+ \quad (\text{Christ suffered for finite duration})$$

$$t_C < \infty \quad (\text{Any real number is less than infinity})$$

Plain English: The penalty is eternal (infinite time), but Christ's suffering lasted three days (finite time).

Theological Response Formalized:

Christianity cannot claim $t_C = t_P$ directly (since $3 \neq \infty$), so it proposes that Christ's infinite value compensates for finite duration. This assumes some function exists:

$$F(v_C, t_C) = t_P$$

where F makes finite suffering equivalent to infinite punishment.

Most commonly proposed: $v_C \times t_C = t_P$ where $v_C = \infty$ (Christ's person has infinite worth)

Why This Fails—Three Independent Arguments:

Argument 1: Dimensional Incompatibility

Duration and value have different dimensions (time vs. worth).

In dimensional analysis: $[\text{worth}] \times [\text{time}] \neq [\text{time}]$

Plain English: You can't multiply dollars by hours to get hours—they're different types of quantities.

In physics, dimensional analysis prevents adding incompatible units:

- 5 meters + 3 seconds = nonsense (different dimensions)
- 5 meters \times 3 seconds = 15 meter-seconds (new dimension, not meters)

Similarly here:

- Penalty specifies: separation for ∞ time
- Payment offers: separation of ∞ value for 3 days
- Result: $\infty \times 3 \text{ days} = \infty \text{ value-days}$ (wrong dimension)

The penalty requires infinite temporal extension. No multiplication by value converts finite time into infinite time—it creates a different quantity entirely.

Argument 2: Mathematical Non-Equivalence

Even granting dimensional compatibility for sake of argument:

$$\begin{aligned} v_C \times t_C &= \infty \times 3 \text{ days} \\ &= \infty \text{ (in value-time units)} \\ &\neq \infty \text{ (in pure time units)} \\ &\neq t_P \end{aligned}$$

Plain English: Infinite value times three days doesn't equal infinite days—it equals infinite "value-days," a fundamentally different quantity.

More formally, for any finite $k > 0$ and any value v (even if $v = \infty$):

$$k \neq \lim_{n \rightarrow \infty} n$$

Therefore: $t_C \neq t_P$ regardless of value assignments.

Argument 3: Penalty Specification

The penalty specifies duration itself as the essential requirement:

$$\text{Penalty} = \{\text{conscious separation from God for time } t : t = \infty\}$$

Duration is not a placeholder for some other quantity. The penalty *is* the infinite temporal extension of conscious separation.

No substitution function F exists where:

$$F(\text{finite duration}) = \text{infinite duration}$$

This would require: $F : \mathbb{R}^+ \rightarrow \{\infty\}$, which is undefined (finite numbers cannot map to infinity through standard operations).

Conclusion: The payment fails because 3 days \neq eternity, regardless of the payer's worth.

□

The Categorical Difference Problem

Beyond duration, the penalty and payment differ categorically:

Categorical Difference Formalized:

Let S_P = state of eternal conscious separation (the penalty)

Let S_C = state of temporary separation with guaranteed reunion (Christ's experience)

Key properties that differentiate these states:

Property	Eternal Penalty (S_P)	Christ's Experience (S_C)
Duration	Infinite	Finite (3 days)
Knowledge	Knows it's permanent	Knows it's temporary
Hope	Zero (separation is forever)	Present (reunion guaranteed)
Psychological state	Despair of permanence	Endurance with purpose
Outcome	No resolution	Resurrection, glorification

Incomparable suffering: Why Christ's finite experience cannot substitute for infinite penalty.

These are categorically different psychological and experiential states. Even if

$\text{Duration}(S_C) = \text{Duration}(S_P)$ (which it demonstrably doesn't), we would still have $S_C \neq S_P$ due to qualitative differences.

Plain English: Christ experienced temporary separation with full knowledge of coming resurrection and victory. The damned experience permanent separation with no hope of resolution. These are fundamentally different experiences—like the difference between fasting (temporary, chosen deprivation with end in sight) and starvation (permanent, involuntary suffering without hope).

This establishes the basic incommensurability. The serious theological debate concerns whether a more sophisticated account of satisfaction and penance can resolve these problems. We now turn to the strongest philosophical defense of PSA.

The Sophisticated Response: Swinburne's Penance and Reparation Model

The most rigorous contemporary defense of substitutionary atonement comes from Richard Swinburne's *Responsibility and Atonement* (Oxford: Clarendon Press, 1989). Unlike popular apologetics that simply assert "infinite value compensates for finite duration," Swinburne develops a sophisticated framework of penance, reparation, and vicarious satisfaction.

Swinburne's Central Argument

Swinburne distinguishes four components of adequate response to wrongdoing:⁹

1. **Repentance:** Recognition of wrong and commitment to avoid repetition
2. **Apology:** Expression of sorrow to the wronged party
3. **Reparation:** Making good the harm done
4. **Penance:** Costly act demonstrating sincerity

For sin against God, humans face an *incommensurability problem*: finite creatures cannot provide adequate reparation or penance to an infinite God. No finite action can repair infinite offense.

Swinburne's solution: Christ's sacrifice provides perfect penance and reparation *on behalf of humanity*. The atonement works not through strict penal equivalence but through:

1. **Identification:** Christ identifies with sinful humanity through incarnation
2. **Perfect Offering:** As God-man, Christ's sacrifice has supreme value

⁹Swinburne, *Responsibility and Atonement*, 81-91.

3. **Divine Acceptance:** God accepts this offering as sufficient for human reconciliation
4. **Human Appropriation:** Individuals appropriate this sacrifice through repentance and faith

Plain English: Think of a family member who pays your debt to restore your relationship with someone you've wronged. The creditor accepts the payment not because it's mathematically equivalent, but because it satisfies the relational requirements—the debt is acknowledged, reparation is made by someone authorized to act on your behalf, and reconciliation becomes possible.

How This Attempts to Resolve the Incommensurability

Swinburne argues the basic critique misunderstands the mechanism:¹⁰

1. Not Strict Quantitative Exchange

The atonement isn't a mathematical equation ($3 \text{ days} \times \infty \text{ value} = \infty \text{ days}$). Rather, it's a *qualitatively perfect offering* that God chooses to accept as sufficient.

Formally: Let A = adequacy of atonement. Swinburne proposes:

$$A = f(\text{Perfect Penance, Divine Acceptance, Union with Humanity})$$

where adequacy derives from *relational satisfaction* rather than mathematical equivalence.

2. Supererogatory Merit

Christ's sinless life accumulated "supererogatory merit"—moral worth beyond what was required of Him. This surplus can be transferred to humanity.¹¹

Analogy: Someone who exceeds all obligations builds up moral "capital" they can gift to others. Christ's perfect obedience created infinite merit transferable to believers.

3. Vicarious Satisfaction Through Union

Christ's hypostatic union (divine-human nature) allows Him to act as humanity's representative. His satisfaction of divine justice counts as humanity's satisfaction through mystical union.¹²

Formally:

Christ \in Humanity	(Incarnation)
Christ's Merit = ∞	(Divine nature)
Therefore: Humanity has access to ∞ merit	(Union)

¹⁰Ibid., 148-162.

¹¹Ibid., 154-156.

¹²Ibid., 157-159.

4. Divine Prerogative

Ultimately, God determines what counts as sufficient satisfaction. If God accepts Christ's sacrifice as adequate, then it *is* adequate—not because mathematics demands it, but because divine prerogative establishes it.¹³

Charitable Assessment: What Swinburne Gets Right

Before critiquing, we must acknowledge the sophistication:

1. **Recognizes the problem:** Unlike simple appeals to "infinite value," Swinburne acknowledges finite/infinite incommensurability
2. **Develops alternative framework:** Shifts from penal equivalence to penance/reparation model
3. **Philosophical rigor:** Draws on philosophy of action, ethics, and relational metaphysics
4. **Historical grounding:** Connects to patristic and medieval satisfaction theories (Anselm, Aquinas)

If PSA can be defended philosophically, Swinburne's approach represents the most careful attempt. His framework doesn't rely on crude "value times duration" calculations but develops a nuanced theory of vicarious satisfaction.

Why the Sophisticated Defense Fails: Four Decisive Problems

Despite its philosophical sophistication, Swinburne's framework faces fundamental difficulties that cannot be resolved through further refinement.

Problem 1: The Substitution Incoherence

The Core Difficulty:

Penance and reparation are *essentially non-transferable*. The logic of moral responsibility requires that the wrongdoer perform these acts. Vicarious penance is a category error—like vicarious contrition or vicarious repentance.

Formal Analysis:

Let M = moral obligation, A = agent who committed wrong, B = proposed substitute

¹³Ibid., 160-162.

For genuine penance:

Penance_A satisfies M_A iff A performs the penitential act

If B performs the act:

$\text{Act}_B \not\rightarrow \text{Penance}_A$

Plain English: If I insult you, my apology has moral significance. If someone else apologizes on my behalf while I remain unrepentant, this doesn't satisfy the moral requirement. The apology must come from the wrongdoer.

Swinburne's Response (Anticipated):

Swinburne argues penance *can* be vicarious if:¹⁴

1. The wrongdoer endorses the substitute's action
2. The substitute is authorized to act on wrongdoer's behalf
3. The wronged party accepts the vicarious action

Why This Fails:

Even granting these conditions, vicarious penance introduces a fatal asymmetry:

The Accountability Transfer Problem:

If Christ's suffering satisfies the penalty, then moral accountability transfers from sinner to Christ. But moral accountability is *constitutive of personhood*—it cannot be transferred without transferring the person.

Formally:

If: $\text{Accountability}(A, \text{Sin}) \rightarrow \text{Penalty}(A)$

And: Christ bears Penalty

Then: $\text{Accountability}(A, \text{Sin}) \rightarrow \text{Penalty}(\text{Christ})$

This requires: $\text{Accountability}(A, \text{Sin}) = \text{Accountability}(\text{Christ}, \text{Sin})$

But this is impossible without $A = \text{Christ}$ (identity of persons). Accountability is a person-indexed property—my accountability for my actions cannot become your accountability without metaphysical identity transfer.

The Financial Debt Disanalogy:

Swinburne's framework relies heavily on debt-payment analogies. But these fail because:

Financial debt: Fungible (transferable)

- Bank doesn't care *who* pays the mortgage, only that it's paid

¹⁴Swinburne, *Responsibility and Atonement*, 152-154.

- Money is impersonal medium of exchange
- No essential connection between debtor and payment source

Moral debt: Non-fungible (non-transferable)

- Justice requires the *wrongdoer* makes amends
- Moral repair is essentially personal
- The identity of who performs penance matters constitutively

Example: If someone murders your family member, justice is not satisfied if their wealthy friend serves their prison sentence while they go free. The penalty is intrinsically tied to personal accountability.

Swinburne's error is treating moral accountability like financial debt—but the fungibility that makes vicarious payment coherent for money makes it incoherent for moral responsibility.

Problem 2: The Divine Acceptance Arbitrariness

The Fundamental Issue:

Swinburne's framework ultimately collapses into divine voluntarism: "It works because God says it does." This evacuates the atonement of rational structure, making God's justice arbitrary.

The Dilemma:

Either:

1. There are objective moral principles governing satisfaction

- Then: These principles must be coherent and apply consistently
- Problem: Vicarious satisfaction violates principle that wrongdoer makes amends
- Result: God cannot consistently accept Christ's sacrifice as satisfying justice

2. God's acceptance makes something adequate by fiat

- Then: No inherent reason why Christ's sacrifice is necessary
- Problem: God could equally accept a sparrow's death, a sincere apology, or unilateral forgiveness
- Result: The entire atoning mechanism becomes arbitrary—why crucifixion specifically?

Swinburne's Horn:

Swinburne attempts to navigate this by claiming God's acceptance is *both* free and *fitting*—Christ's sacrifice is supremely appropriate, and God freely chooses to accept it.¹⁵

But this doesn't escape the dilemma:

If Christ's sacrifice is *objectively fitting* for satisfaction, then its fittingness derives from principles of justice independent of divine will. These principles must explain why vicarious suffering satisfies—but no such principles exist that preserve moral coherence (per Problem 1).

If fittingness is just God's *aesthetic preference*, then divine acceptance is arbitrary after all—God could have chosen any number of alternatives and declared them "fitting."

The Abelardian Collapse:

This problem drives sophisticated thinkers toward Abelard's moral influence theory: Christ's death demonstrates God's love, transforming human hearts—not satisfying objective justice requirements.

But this abandons PSA entirely. If the cross functions through moral influence rather than satisfaction of penalty, then:

- No payment actually occurs
- Justice isn't satisfied, just waived
- The entire penal framework collapses

Swinburne's framework, pressed hard enough, slides into either arbitrariness (horn 2) or abandons substitutionary satisfaction altogether.

Problem 3: The Supererogatory Merit Incoherence**The Conceptual Problem:**

Swinburne's framework requires Christ accumulate "supererogatory merit" transferable to humanity. But this concept is incoherent when applied to a divine person.

Formal Analysis:

Supererogation requires:

1. Actions beyond what duty requires (going beyond obligation)
2. Possibility of not performing supererogatory acts without moral failure

Formally: $\text{Supererogatory}(A) \equiv [\text{Good}(A) \wedge \neg \text{Obligatory}(A)]$

Applied to Christ:

¹⁵Swinburne, *Responsibility and Atonement*, 160-161.

If Christ is God incarnate:

$$\begin{aligned} \text{Divine Nature}(C) &\rightarrow \text{Morally Perfect}(C) \\ \text{Morally Perfect}(C) &\rightarrow \forall x : \text{BestAction}(x) \rightarrow \text{Obligatory}_C(x) \end{aligned}$$

Plain English: If Christ is God, He cannot fail to do what is best. But if He cannot fail to do what is best, then doing what is best is obligatory for Him—not supererogatory.

The Dilemma for Divine Supererogation:

Either:

1. Christ's perfect obedience was required by His nature

- Then: Not supererogatory (was obligatory)
- Result: No surplus merit to transfer

2. Christ's perfect obedience was optional

- Then: Christ could have sinned or failed
- Result: Contradicts divine impeccability (returns to Hypostatic Union problems)

The Human Nature Escape Attempt:

One might argue: "Christ's obedience was supererogatory *qua* human nature—humans aren't obligated to be sinless."

But this faces two problems:

1. Person-level accountability: As established in Hypostatic Union critique, moral properties attach to persons, not nature-fragments. The question "Was Christ obligated to be sinless?" asks about the person Christ, not Christ-qua-something.

2. Divine nature dominance: If Christ's divine nature is truly present, it would render sin metaphysically impossible—making sinlessness necessary, not supererogatory. You can't accumulate supererogatory merit for doing what you cannot fail to do.

The Broader Incoherence:

The very concept of "transferable moral merit" is dubious. Moral worth is achievement-indexed—its value derives partly from *who* performed the act under *what circumstances*.

If I donate to charity, the moral worth is mine because *I* chose to overcome selfishness, *I* prioritized others' needs, *I* demonstrated virtue. This worth cannot be transferred to you like a commodity—that would sever moral worth from its constitutive connection to agency.

Swinburne's framework requires moral merit function like currency in a ledger (accumulate here, transfer there), but moral worth doesn't have this structure.

Problem 4: The Incommensurability Returns**The Devastating Realization:**

Even granting everything in Swinburne's framework—vicarious penance, divine acceptance, supererogatory merit—the *original incommensurability problem remains unsolved*.

Why the Problem Persists:

Recall the penalty specification:

$$P = \{\text{conscious separation from God for duration } t : t = \infty\}$$

The penalty isn't "punishment of equivalent value" or "adequate penance of some kind"—it's specifically *eternal conscious separation*.

Christ's sacrifice provides:

$$C = \{\text{conscious separation from God for duration } t : t = 3 \text{ days}\}$$

Swinburne's framework doesn't bridge this gap—it just *declares* the gap bridged through divine acceptance.

The Formal Problem Remains:

For any proposed equivalence function F :

$$F(C) = P \implies F(\text{finite duration separation}) = \text{infinite duration separation}$$

No matter how we modify F with "perfect penance," "supererogatory merit," or "divine acceptance," we still have:

$$F : \mathbb{R}^+ \rightarrow \{\infty\}$$

This is mathematically undefined. You cannot construct a function that maps finite inputs to infinity through any standard operation.

The Substitution Shell Game:

Swinburne's sophistication involves replacing *what needs to be equivalent*:

Original claim: Christ's suffering \equiv eternal punishment

Swinburne's substitution: Christ's offering \equiv adequate satisfaction

But "adequate satisfaction" is now defined simply as "what God accepts," which returns us to Problem 2 (divine voluntarism / arbitrariness).

The incommensurability wasn't resolved—it was *obscured* through conceptual redefinition. When we ask "But why does finite suffering satisfy infinite penalty?" the answer is ultimately just "Because God says so"—which is no answer at all.

The Categorical Difference (Remains):

Even if duration could somehow be satisfied, the categorical differences in Table ?? remain:

Christ knew His suffering was temporary, purposeful, and would end in resurrection and glorification. The damned experience permanent, hopeless separation.

These are qualitatively different states. No amount of theological sophistication makes temporary-with-hope equivalent to permanent-without-hope. They're different experiential categories.

Additional Inadequate Responses

Having examined and refuted the strongest philosophical defense, we can treat other common responses more briefly.

The "Infinite Value" Claim

Defense: "Christ has infinite value as God, so His finite suffering has infinite worth."

Problems:

1. **Dimensional incompatibility** (already demonstrated): $[\text{value}] \times [\text{time}] \neq [\text{time}]$
2. **Value doesn't convert duration:** High-value suffering doesn't become longer-duration suffering—it remains finite
3. **Person vs. action confusion:** Even if Christ's *person* has infinite value, His *actions* occur in finite time

Plain English: A billionaire serving 3 days in jail doesn't serve more time than a pauper serving 3 days. Their value doesn't convert into temporal duration.

The "Intensity Compensates" Claim

Defense: "Christ suffered the equivalent intensity of eternal torment compressed into 3 days."

Problems:

1. **Intensity \neq duration:** Greater intensity per unit time doesn't equal more time
2. **Physical limits:** Human bodies (which Christ had) cannot experience infinite intensity—they have pain thresholds and would lose consciousness or die
3. **Still finite:** High intensity \times Finite time = Finite total suffering (still $< \infty$)

Mathematically: $\lim_{I \rightarrow \infty} (I \times 3 \text{ days}) = \infty$ (in intensity-time units), but $\neq \infty$ (in pure time units).

The "Qualitative vs. Quantitative" Claim

Defense: "God cares about quality of suffering, not quantity. Christ's perfect suffering is qualitatively adequate."

Problems:

1. **Penalty specification:** Scripture specifies eternal *duration* ("eternal punishment," "forever and ever," "where their worm does not die")—this is quantitative
2. **Shifts goalposts:** If quality matters more than quantity, why is ECT eternal? Why not brief but intense purification?
3. **Arbitrary standards:** Who determines "qualitative adequacy" and by what measure?

This defense implicitly abandons the very doctrine it's defending (eternal punishment) by suggesting duration doesn't matter.

The "Mystery" Appeal

Defense: "How Christ's finite suffering satisfies infinite penalty is a divine mystery."

Problem:

Mystery must mean "I don't understand the mechanism" (epistemic limitation), not "it violates mathematics" (logical impossibility).

We don't appeal to mystery when asked: "Can God create a married bachelor?" or "Can God make $2 + 2 = 5$?" These aren't mysteries—they're incoherent.

Similarly, $3 \text{ days} = \infty$ isn't mysterious—it's false. No appeal to divine mystery makes mathematical impossibilities possible.

Conclusion: Why PSA Cannot Be Salvaged

We have examined three types of responses to the PSA incommensurability:

1. **Naïve defenses** (infinite value, intensity compensation, qualitative vs. quantitative) fail on basic mathematical and dimensional grounds.
2. **Sophisticated defenses** (Swinburne's penance/reparation framework) address the problem seriously but face four insurmountable difficulties:
 - Penance is essentially non-transferable (substitution incoherence)
 - Divine acceptance becomes arbitrary (voluntarist collapse)
 - Supererogatory merit is incoherent for divine persons

- The duration incommensurability remains unsolved (just obscured through redefinition)
3. **Evasive strategies** (mystery appeals) abandon rational discourse without resolving the problem.

Implications Beyond PSA:

The failure of PSA undermines central Christian claims:

- **Soteriology collapses:** If Christ's death doesn't satisfy penalty, what does it accomplish?
- **Divine justice questioned:** If God punishes infinitely for finite sins, yet accepts finite satisfaction from Christ, justice becomes incoherent
- **Systematic theology fractures:** Atonement connects Christology, hamartiology, soteriology, and eschatology—its failure destabilizes the system

The Mathematical Certainty:

Unlike some theological disputes where reasonable people can disagree, this is mathematical:

$$3 \text{ days} \neq \infty$$

No theological sophistication, divine prerogative, or mystical union changes this. The incommensurability isn't a puzzle to be solved through clever theology—it's a fundamental impossibility revealed through basic mathematics.

Connection to Cognitive Dissonance:

Professional theologians understand dimensional analysis. Apologists know that $\text{finite} \times \text{value} \neq \text{infinite duration}$. Yet they continue teaching PSA as coherent doctrine.

This is precisely the condition that produces Cognitocornus Dissonantia: conscious recognition of incoherence combined with continued advocacy. The horns grow not from mathematical ignorance but from the gap between what is recognized privately and what is maintained publicly.

The first two proofs demonstrate internal contradictions within Christian doctrine—claims that violate logic (Proof I) or mathematics (Proof II). The third proof examines Christianity's defensive structure—how apologetics systematically prevents these contradictions from functioning as falsifying evidence, rendering the belief system immune to correction.