

Deductive Closure

Logical Impossibility, Evidential Support, and the Limits of Apologetic Accumulation

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Central claim

Under classical consequence and stable meanings, a doctrine-set that entails a contradiction has no coherent truth-condition for contingent evidence to support. Evidence may support consistent components or a revised successor, but not the unchanged contradictory conjunction. This conditional rule does not show that Christianity is inconsistent; explicit paraconsistency changes the framework.

Preview:

Claim under review	Logic gate	What evidence can support	Honest next step
A specified Christian doctrine-set is offered as one cumulative target.	First determine whether the same stable claims entail a contradiction in the logic being used.	Consistent components, historical reports, experiences, or a coherent revised target.	Defeat the derivation or disclose the changed premise, meaning, logic, or doctrine; do not credit nearby support to the unchanged contradiction.

The argument begins only after genuine deductive defeat. Apparent contradiction, low probability, and uncertainty about a derivation require different responses.

Visual preview:



Abstract

Christian apologetics often assembles testimony, religious experience, historical claims, moral arguments, and explanatory considerations into a cumulative case. Such accumulation can be rationally important, but it cannot repair a target that genuinely entails a contradiction under fixed meanings and classical consequence. A contradictory doctrine-set is not simply a very improbable hypothesis waiting for a larger likelihood ratio. As stated, it has no coherent truth-condition for contingent evidence to confirm.

This paper develops a deductive-closure rule for evidential argument. It distinguishes logical impossibility from an ordinary prior of zero, uncertainty about whether a contradiction has been derived, and a deliberate move to paraconsistent logic. It then traces how evidence for a coherent component, historical report, or neighboring theological claim can be transferred illicitly to a stronger contradictory conjunction. Worked examples involving a coin, a historical event, motion relative to a frame, a hypothetical knowledge claim, and a larger doctrinal package show which kinds of support survive and which require an acknowledged change of target.

The conclusion is conditional but strict. If deductive defeat is genuine, a cumulative case must defeat the derivation or revise a premise, meaning, logic, or doctrine; additional contingent evidence cannot confirm the unchanged target. This does not establish that Christianity, theism, or any named doctrine is inconsistent. It requires critics to supply an exact derivation and permits defenders to contest it, while preventing mystery, evidential volume, or nearby support from doing the work of coherence.

Keywords: deductive closure; logical contradiction; evidential support; Bayesian confirmation; cumulative apologetics; target revision

1 The Gate Before the Pile

Many apologetic arguments are cumulative. A defender may combine testimony, manuscript history, religious experience, moral realism, cosmic order, personal transformation, and alleged miracles, then ask readers to assess the whole rather than any single strand. Sophisticated versions can make their comparative reasoning explicit. Swinburne develops a Bayesian cumulative case for theism, while McGrew and McGrew construct a detailed cumulative argument for the resurrection (Swinburne 2004; T. McGrew and L. McGrew 2009). Nothing in this paper implies that cumulative reasoning is inherently defective.

The challenge concerns the identity of the target. Evidence bears on propositions under descriptions, not on a devotional package merely because its parts travel together in practice. Confirmation theory has long treated the relation among hypothesis, background assumptions, and evidence as structured rather than free-floating (Hempel 1965; Fitelson 2006). When the target changes, the evidential question changes with it.

By a *God-concept* I mean a specified conception of a deity with identifiable attributes and relations. Generic theism, a historical claim about first-century testimony, a resurrection claim, Nicene Christology, and a denomination's full doctrinal package are not one hypothesis at different volumes. They are different targets connected by additional premises. Evidence for one does not automatically confirm the rest.

The governing claim is narrower still. Suppose a target doctrine-set, under stable meanings and the classical consequence relation used in its defense, entails both a proposition and its negation. Call this *strict deductive defeat*. The paper does not supply such a derivation for Christianity. It asks what follows if one has survived scrutiny. The answer is that ordinary contingent evidence cannot confirm the same unchanged target as true. The dispute must return to the derivation, the semantics, the logic, or the doctrine.

The deductive-closure rule

Once a specified target is strictly deductively defeated under fixed meanings and an agreed consequence relation, no amount of contingent evidence confirms that unchanged target as true. Evidence may undermine the alleged derivation or support a coherent component or successor. Those are rationally important outcomes, but each changes what has been supported.

This is not the claim that coherence is sufficient for truth. Countless coherent hypotheses are false. It is the claim that coherence is a gate for truth-directed confirmation within the chosen framework. A larger evidential pile cannot substitute for passing through it.

2 Four States That Must Not Be Blurred

Apologetic and skeptical discussions often use *impossible*, *incoherent*, *improbable*, and *mysterious* too loosely. The result is false severity by critics and false rescue by defenders. Table 1 separates four importantly different states.

Table 1: A logical-status ledger for evidential claims.

Status	What the status means	What evidence may do	Required response
Coherent and live	At least one admissible interpretation or model satisfies the target.	Raise or lower its standing relative to alternatives.	Specify likelihoods, rivals, dependence, and background assumptions.
Coherent but assigned zero	The target is possible, but an agent or model gives it an ordinary zero prior.	Standard conditionalization preserves the zero; model revision may reopen it.	Explain whether the zero is a revisable policy or a principled limit.
Apparently contradictory	A derivation is alleged, but a premise, meaning, scope, or inference remains contested.	Support a diagnosis about language, premises, or which derivation is credible.	Audit the argument before declaring deductive defeat.
Strictly contradictory	Under fixed meanings and the selected logic, the target entails a claim and its negation.	Support components, reports, or a revised successor, but not the unchanged conjunction as true.	Defeat the derivation or disclose the changed premise, semantics, logic, or target.

This ledger imposes symmetry. Critics do not get to label a difficult doctrine contradictory without a derivation. Defenders do not get to answer a completed derivation with unrelated positive evidence. Both sides must say which state they think the target occupies and why.

The distinction between apparent and strict contradiction is especially important in theology. Terms such as *person*, *nature*, *knowledge*, *freedom*, and *identity* may carry technical meanings. A critic who ignores those meanings may attack a caricature. Yet technical vocabulary is not a blank check. The meanings must be stable enough for affirmation, denial, and consequence. A shift introduced only after contradiction appears may yield a coherent successor while leaving the initially advertised claim undefended.

3 Logical Zero Is Not Just a Stubborn Prior

Bayesian language helps only when used carefully. Standard conditionalization redistributes probability among possibilities represented in a model. A hypothesis assigned prior probability zero remains at zero after ordinary conditioning, provided the update is defined. Modern Bayesian texts make the dependence on a specified probability model explicit (Earman 1992; Howson and Urbach 2006; Sprenger and Hartmann 2019). But two different reasons for zero must be kept apart.

An *epistemic zero* is a judgment within an agent’s model. It may express dogmatism, idealization, a limiting distribution, or a decision to omit a possibility. Such a zero can be criticized. The rational repair may be to use a small positive prior, an imprecise probability, or a richer updating rule. Hájek’s analysis of conditional probability shows why conditioning on zero-probability events is not exhausted by the elementary ratio definition (Hájek 2003). Learning can therefore expose a poor probability model.

A *logical zero* in the present argument is different. It represents an empty target under the fixed meanings and consequence relation: there is no admissible case in which the conjunction is true. Replacing zero with a tiny positive number would not express open-mindedness. It would silently replace logical impossibility with extreme rarity. The repair belongs in the logic, semantics, or target, not in the likelihood ratio.

This distinction answers an obvious worry about logical omniscience. Human reasoners do not instantly know every consequence of their beliefs. Evidence may rationally change our confidence that an alleged contradiction was derived correctly. A newly found manuscript can change which doctrine a community actually affirmed; a semantic analysis can show that two predicates were equivocal; a proof check can reveal an invalid step. These updates bear on the *meta-hypothesis* that strict deductive defeat occurred. They do not make a contradiction true while keeping the derivation, meanings, and logic fixed.

The order of inquiry is therefore important. First assess whether the gate has actually failed. Only then ask which evidence bears on a remaining live target. Treating the gate as one more low-weight item in a cumulative total reverses the dependency.

4 The Nearby-Target Transfer

Evidence often survives deductive defeat, but its target becomes narrower. Reports can remain evidence that witnesses testified. Experiences can remain evidence that subjects underwent powerful states. Social effects can remain evidence that a tradition shaped lives and institutions. Arguments from order can remain relevant to a minimal explanatory hypothesis. None of these evidential relations disappears merely because a larger doctrine-set fails.

The error is to transfer that surviving support back to the defeated conjunction. If a target consists of several claims joined together, evidence that favors one component need not favor the conjunction by the same amount. When another component is contradictory under the framework, the transfer is blocked altogether. Table 2 makes the bookkeeping visible.

Table 2: Evidence may survive while the advertised target does not.

Evidence strand	Nearest live target	What does not follow automatically	Honest continuation
Historical testimony	People reported, remembered, or sincerely believed an event.	Every metaphysical doctrine attached to the report is true.	Test the event claim and each further bridge separately.
Religious experience	Subjects had experiences interpreted as transcendent.	One specified God-concept or a full doctrinal package best explains them.	Compare religious, psychological, social, and mixed explanations.
Moral or cosmic argument	Some feature may favor a minimal explanatory proposal.	The proposal entails incarnation, Trinity, inerrancy, or a church’s complete theology.	State the added premises and test each target at its own level.
Transformative effects	A practice or community produced real change.	The causal theology is true or the transformation uniquely favors it.	Compare mechanisms and separate benefit from doctrinal truth.

The examples are schematic, not verdicts on the cited cumulative cases. Their point is grammatical: support must remain attached to the proposition whose expectedness the evidence changes. A broad label such as “evidence for Christianity” conceals the bridgework.

This is also why evidential volume cannot do the work. Ten strands bearing on one coherent component may provide a formidable case for that component. Multiplying the number of strands does not create an inference from the component to every additional doctrine. Accumulation can strengthen support; it cannot erase missing entailments or supply truth-conditions to an empty conjunction.

5 Worked Examples of Deductive Closure

The closure rule becomes concrete when four questions are kept in order: What is the exact target? Where does the contradiction arise? What proposition does the new evidence actually support? What must change before a live target reappears? Table 3 applies those questions across ordinary, historical, scientific, and theological cases. Every example is schematic. The point is not that each contradiction has been established, but that the evidential options become clear if it has.

Table 3: Worked examples of closure, nearby support, and repair.

Exact target	What evidence may establish	Why it does not confirm the target	Honest repair
Coin at one instant	The coin exists, a photograph is authentic, or one face was visible from one angle.	The target says the same coin is heads-up and not heads-up at the same instant under the same face criterion. More photographs cannot make that conjunction true.	Distinguish coins, times, surfaces, or reflections; expose an image error; or withdraw one claim.
Battle at a fixed place and date	A source reported a battle, artifacts indicate conflict nearby, or one chronology is credible.	Those facts may favor one report, but they do not confirm that the battle both occurred and did not occur at the same place and date.	Correct the date or location, reject one report, or show that the negative claim meant something narrower.
Motion in one reference frame	Instruments recorded positions, a sensor was miscalibrated, or two observers used different frames or tolerances.	Once frame, time, and criterion are fixed, the exact target cannot be both at rest and moving in that same respect.	Distinguish frames, times, or tolerances; repair the measurement; or abandon one description.
Hypothetical knowledge claim	Testimony, religious experience, or historical evidence supports a neighboring theological claim.	Such evidence does not confirm that one subject both knows a truth and lacks knowledge of it at the same time and in the same sense.	Reject a premise; distinguish times, respects, natures, or senses of knowledge; or explicitly change the logic.
Larger doctrinal package	Evidence strongly supports a historical component, moral argument, practice, or experience within the package.	Support for a live component does not supply a truth-condition to a separate contradictory component of the conjunction.	Preserve the supported components while defeating, revising, or removing the contradictory part.

5.1 More witnesses do not alter a contradictory target

Suppose a claim says that one coin is heads-up and not heads-up at the same instant, using the same surface and the same meaning of “heads-up”. A hundred independent photographs may establish that the coin was present, that the cameras worked, and that observers reported what they saw. The images may also reveal that one photograph shows a reflection, the underside, another coin, or a later instant. Each discovery can defeat the alleged contradiction by changing our description of the case. What the photographs cannot do is leave every term fixed and make the contradiction better supported. They either select between the claims or show that the original target was misdescribed.

The historical example works the same way. Archaeological remains may support a battle near a city, while textual evidence may favor one date over another. That is real evidential progress. Yet if the

advertised target still says that the same battle both occurred and did not occur at the same place and date, the evidence has not confirmed that conjunction. It has instead supported a report, resolved a chronology, or motivated a revised target. Calling all three outcomes “support for the original account” hides the change.

5.2 Clarification can reopen a live target

An object may be at rest relative to a train and moving relative to the ground. There is no contradiction because the reference frames differ. Likewise, an instrument may classify motion below a stated tolerance as “at rest” while a more precise instrument records a small displacement. Again, the predicates differ in operational meaning. These are not evasions when the frames and tolerances are independently specified and consistently used. They show why critics must stabilize meanings before announcing deductive defeat.

But suppose the frame, time, tolerance, and meaning are all fixed, and the target still says that the object is exactly at rest and exactly moving. More measurements may reveal which claim failed or whether an instrument malfunctioned. They cannot confirm both descriptions as true in that same respect. Evidence can reopen the gate by defeating the derivation; it does not pass through a gate that remains closed.

5.3 A hypothetical theological case

Consider a purely hypothetical doctrine-set containing three claims:

1. the same subject knows every truth at a specified time;
2. a proposition is true at that time; and
3. the subject does not know that proposition at that time, where *knows* has the same sense and respect throughout.

Under those fixed conditions, the set entails both that the subject knows the proposition and that the subject does not know it. Testimony, religious experience, historical evidence, or an argument from cosmic order may support other claims associated with the tradition. None confirms this unchanged conjunction. The legitimate responses are to deny one premise, show that the time or sense differs, supply an independently motivated distinction of respect or nature, or adopt and defend a nonclassical logic. This example does not allege that any Christian doctrine has this form. It shows exactly where evidence can bear: on whether the derivation is accurate, on neighboring claims, or on a revised successor.

5.4 A cumulative package may survive in parts

Finally, imagine a package containing a historical claim, a moral argument, a report of religious experience, and a separate pair of strictly contradictory propositions. Suppose the first three components receive unusually strong evidence. Deductive closure does not erase that evidence. The historical claim may become very credible, the moral argument may remain forceful, and the experience report may be well attested. What does not follow is that their combined support flows into the contradictory pair and thereby confirms the entire conjunction.

The rational result is preservation without transfer. Keep every component that remains live and supported. Then defeat the contradiction, revise the package openly, or narrow the conclusion to what the evidence actually warrants. This is less destructive than treating the package as all-or-nothing, but more disciplined than allowing a common label to move support across a logical gap.

6 Mystery, Redefinition, and Explicit Revision

Mystery can be an intellectually responsible response. A defender may have reason to believe that an apparent conflict reflects incomplete understanding rather than a true contradiction. The deductive-closure rule allows that reply because strict defeat has not yet been established. What mystery cannot do is preserve both a completed contradiction and the ordinary evidential claims attached to the same meanings.

Redefinition is likewise not automatically evasive. Philosophical and theological inquiry often improves concepts. A technical distinction between person and nature, for example, may show that two predicates do not attach to one subject in the same respect. If the distinction is independently motivated and applied consistently, it may defeat the contradiction. The correct description is then that the derivation failed under the clarified target.

The cost must nevertheless be disclosed. If a term carried one ordinary content when positive evidence was presented and a different technical content when contradiction threatened, the evidence may not transfer to the revised claim. A coherent successor inherits support only when the evidential relation survives the change of meaning. Revision is not defeat, but neither is it costless continuity.

The practical fork is simple:

1. defeat the alleged contradiction while retaining the target;
2. revise a premise or meaning and reassess the evidence for the successor;
3. change the consequence relation and defend that logic; or
4. abandon the defeated conjunction while preserving whatever components remain supported.

Each route can be rational. The closure rule excludes only a fifth route: leave the contradiction, logic, and meanings untouched while asking additional contingent evidence to make the target confirmable.

7 Strong Objections and Their Costs

7.1 Evidence can reveal that logic or meaning was misunderstood

Yes. Evidence can bear on historical usage, authorial intent, linguistic practice, and which formalization represents a doctrine. That is evidence against the claim that strict deductive defeat was established. It is not evidence that raises the probability of the contradiction while the formalization remains fixed. The objection reinforces the need for a proof audit before closure.

7.2 Zero priors should never be absolute

As a rule for ordinary empirical inquiry, resistance to dogmatic zero can be wise. A surprising observation should sometimes prompt model expansion. But model expansion changes the space of candidates. The deductive-closure rule concerns a structural zero generated by an empty truth-condition, not an agent's refusal to consider a live possibility. Replacing the structural zero with a small number assumes the very revision that must be stated.

7.3 Useful scientific theories have contained inconsistencies

Scientists can reason productively with partially inconsistent information. One may isolate the conflict, restrict inferences, use approximations, or retain a model instrumentally while seeking repair. This shows that inconsistency need not make a body of sentences useless. It does not show that the inconsistent conjunction is classically true. Pragmatic usefulness, component reliability, and approximate success are nearby targets, not confirmation of the whole as an exception-free description.

7.4 Some contradictions may be true

Dialetheism rejects the assumption that no contradiction is true, and paraconsistent logics block the classical explosion from a contradiction to arbitrary conclusions. Priest provides the major systematic defense of this position (Priest 2006). Beall applies a contradiction-tolerant framework directly to Christology, offering a strong theological alternative rather than merely invoking mystery (Beall 2021).

This objection identifies the paper’s principal boundary. The deductive-closure rule is conditional on the consequence relation in which the defeat is derived. An explicitly paraconsistent theology can reject classical closure, but it must then state its logic, truth theory, and permitted inferences. Ordinary Bayesian or classical apologetic accumulation cannot quietly borrow that escape while continuing to reason as though contradictions were fatal everywhere else.

7.5 A worldview is confirmed holistically, not sentence by sentence

Holism can explain why evidence bears on networks rather than isolated claims. It does not eliminate target identity. A network must still specify what counts as its acceptance, which revisions preserve it, and which observations discriminate it from rivals. If any internal change is treated as the same worldview, the label becomes too elastic to carry the advertised conclusion. Holistic confirmation needs disciplined revision rules, not exemption from them.

8 A Six-Step Closure Audit

The argument becomes useful when it disciplines both critics and defenders. Before a cumulative case is weighed against a deductive objection, conduct the following audit.

1. **Name the target.** State the exact doctrine-set rather than using “Christianity” or “theism” as a container for changing claims.
2. **Stabilize meanings and logic.** Specify the senses of contested terms, relevant scopes and respects, and the consequence relation governing the derivation.
3. **Display the alleged defeat.** Identify the premises and the precise proposition and negation said to follow.
4. **Test the derivation.** Consider equivocation, invalid inference, hidden modal shifts, inconsistent source attributions, and independently motivated distinctions.
5. **Partition the evidence.** Record which live component, report, experience, or successor each strand bears on; do not use a package label as an inferential bridge.
6. **Disclose the repair.** If the target survives, say why. If it changes, name the changed premise, meaning, logic, or doctrine and reassess support for the successor.

The closure discipline

Coherence is not a score added to testimony, experience, or explanatory appeal. It determines which target those forms of evidence can address. Critics owe a valid derivation; defenders owe a stable target. Only after both obligations are met does accumulation have a proposition to confirm.

The audit does not favor skepticism by definition. A defender who defeats an invalid contradiction has strengthened the case by restoring a live target. A critic who concedes that only a weaker component failed has narrowed the objection responsibly. The point is to prevent positive evidence and deductive pressure from passing each other while both sides continue to use the same package name.

9 Conclusion

Contingent evidence can make a coherent hypothesis more or less credible. It can also change our view about whether a contradiction was correctly derived. What it cannot do, under fixed meanings and classical consequence, is confirm an unchanged conjunction that genuinely entails both a proposition and its negation.

The strongest reason is target discipline. Across the coin, battle, motion, hypothetical knowledge, and doctrinal-package examples, evidence either challenges the alleged contradiction, supports a live component, or motivates a revised successor. Historical testimony, religious experience, moral reflection, cosmic order, and transformative effects may therefore retain evidential force. That force does not cross a logical gap merely because the claims belong to one tradition. Evidential volume strengthens what the evidence actually bears on; it does not manufacture coherence.

The principal limit remains decisive. This paper does not prove Christianity, theism, or any named doctrine inconsistent. It tells both sides what follows if strict deductive defeat survives an exact audit. Critics must show the derivation. Defenders must defeat it or disclose the revision. Readers should refuse the fifth route in which a contradiction is left untouched beneath an ever larger evidential pile.

A Formal Reconstruction of the Central Thesis

The central inference can be reconstructed without claiming that its antecedent has been established for any Christian doctrine.

- P1.** Truth-directed contingent evidence can confirm a target as stated only if that target has at least one admissible case in which it is true under the fixed meanings and consequence relation used in the inquiry.
- P2.** If a target strictly entails both a proposition and its negation under those fixed meanings and that consequence relation, it has no such admissible case.
- P3.** Therefore, if strict deductive defeat is genuine, contingent evidence cannot confirm the unchanged target as true.
- P4.** Evidence may instead bear on whether the derivation is sound, on a coherent component of the target, or on a successor produced by revising a premise, meaning, logic, or doctrine.
- P5.** Support for one of those alternatives is not support for the unchanged strictly defeated conjunction unless an additional valid bridge restores a live target.
- P6.** Therefore a cumulative apologetic case facing genuine deductive defeat must defeat the derivation or disclose and defend the revision; evidential accumulation alone cannot close the gap.
- P7. Conclusion.** Under the stated classical and semantic conditions, strict deductive defeat closes the unchanged target to contingent confirmation while leaving exact proof criticism, coherent components, and explicitly revised successors open for assessment.

Validity. P1 and P2 entail P3: a target with no admissible case in which it is true cannot be truth-directedly confirmed as stated. P4 and P5 identify the live evidential alternatives, and P6 applies that distinction to a cumulative apologetic case. The conclusion follows under the stipulated classical consequence relation and stable meanings.

Premise support and scope. P1 and P2 express the conceptual framework defended in the paper. Whether any actual doctrine satisfies their antecedent is a separate interpretive and proof-dependent question. A critic must specify the target and produce a sound derivation; a defender may challenge a premise, meaning, scope, or consequence relation. The reconstruction therefore establishes the result of genuine strict defeat, not that Christianity or any named doctrine has suffered it.

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