

The Valuation Problem in Justice Theory: Incommensurability and the Categorical Failure of PSA — 35

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Central claim

The target account makes endless duration part of the penalty, treats Christ’s passion as finite, and declares the penalty satisfied because of Christ’s infinite worth or status. Its own completion rule is the benchmark: worth may explain significance or acceptance, but it does not by itself supply time undergone or a rule by which another performance counts as completion. The bare worth appeal therefore fails unless the account revises the penalty, justifies an equivalence rule, or identifies another mechanism; this conditional result does not refute every penal-substitution account, doctrine of hell, or theory of atonement.

Preview:

Target account	Completion-rule test	Missing bridge	Result and live alternatives
Endless duration helps define the penalty; Christ’s passion is finite; worth or status is invoked to claim satisfaction.	If duration is constitutive, completion requires that duration or a justified rule explaining why another performance counts.	Worth may explain significance, representation, or acceptance; none by itself is time undergone or an equivalence rule.	The bare worth appeal does not establish satisfaction; revise the penalty, justify equivalence, or name another mechanism.

The test applies to one claim package, not every Christian account. Representation can authorize a bearer, but it does not by itself say what completes the sanction.

Visual preview:



The blue hourglass represents a penalty whose identity includes endless duration; the gold crown, cross, and finite hourglass represent a finite but exceptionally worthy substitute. The unplaced keystone marks the missing equivalence rule, while the open gate marks pardon or commutation as a different route. The image shows that the bare worth bridge is incomplete—not that every bridge or every penal-substitution model is impossible.

Abstract

This paper evaluates a specific form of penal substitutionary atonement: the penalty is endless conscious punishment; endless duration is constitutive of that penalty; Christ's passion is finite; and infinite worth or status is offered as the reason the passion satisfies the penalty. The bare worth appeal does not establish satisfaction because the account's own completion rule has changed without justification. The paper separates three questions often collapsed—who may bear a sanction, what kind of sanction is imposed, and what counts as completing it—and uses work on value comparison, punishment, mercy, and contemporary atonement theory to test the missing bridge. If duration is constitutive, completion requires that duration or a defended equivalence rule; worth, dignity, intensity, and status are not by themselves time undergone. A defender may deny that duration is constitutive, justify an equivalence rule, or identify another completion or release mechanism such as qualitative completion, commutation, or pardon. Representation can answer the bearer question, but it still needs an account of completion. The result is bounded: the Valuation Fallacy defeats one value-for-duration inference, not every form of penal substitution, doctrine of hell, or theory of atonement. The constructive demand is simple: apply one completion rule to the penalty and its alleged satisfaction, or explain the principled change.

Keywords: penal substitutionary atonement; eternal punishment; incommensurability; satisfaction; commutation; punishment; value theory

1. The Completion Rule Under Pressure

Penal substitutionary atonement (PSA) is not one proposition. Accounts differ over the wrong addressed, the nature of punishment, the basis of representation, the role of divine justice, and what Christ's passion accomplishes. Defenders can emphasize punishment in the place of sinners, representative union, legal imputation, sacrificial offering, satisfaction, pardon, or some combination. Recent philosophical and theological treatments display this variety rather than one uncontested mechanism (Craig, 2020; Crisp, 2020; Murphy, 2021). The question here is narrower: when an account uses endless duration to define the penalty, may it abandon duration when explaining how that same penalty was completed? The answer defended here is no—not without an additional rule or a stated change of model.

A *God-concept* is a specified conception of a deity. The target here belongs to a Christian God-concept and combines four commitments:

1. the just penalty for sin is endless conscious punishment;
2. endlessness is not merely a later consequence but part of what makes that penalty the penalty it is;
3. Christ's passion is finite in duration; and
4. the passion nevertheless satisfies the endless penalty because the substitute possesses infinite worth, dignity, or status.

The first commitment has sophisticated defenders, and not all describe damnation in exactly the same way (Walls, 1992). The third is fixed by the target argument: whatever chronology one assigns to the passion, it is not an endless temporal interval. The fourth does not follow from the first three. The contested work lies between the second and fourth commitments. If duration helps constitute the penalty, how does worth satisfy that condition?

Large words cannot do the work of a missing argument. "Infinite" applied to worth and "endless" applied to duration do not automatically name quantities on one scale. A library of immeasurable cultural value does not thereby occupy infinite physical space. An infinitely dignified officeholder does not thereby serve an endless prison term. The question is not whether Christ matters infinitely within a theological scheme. The question is whether worth, status, or dignity is the property the alleged sanction requires.

The completion-rule test follows: apply the same rule when stating what sinners owe and when explaining what the substitute accomplished, unless a principled change is made explicit. If endless duration is constitutive, a finite passion does not meet that condition merely by having unsurpassable worth. Three routes remain open:

argue that duration was never constitutive; justify an equivalence rule explaining why another performance counts as completion; or identify a different completion or release mechanism, such as qualitative completion, commutation, pardon, or a non-penal account. Representation may answer who can act for whom, but it does not by itself answer what completes the penalty. What is blocked is the unargued slide from high worth to completed time.

2. Three Questions That Must Be Kept Separate

The completion-rule test works only if three questions remain separate. Who may bear a sanction, what sanction is imposed, and what completes that sanction are distinct matters. Table 1 states the diagnostic that controls the rest of the paper.

Question	What it asks	What could answer it	What does not yet answer it
Bearer	Who may undergo, represent, or answer for the wrong?	Personal liability, consent, representation, union, imputation, or institutional standing.	Showing that the bearer is exceptionally worthy.
Penalty kind	What response counts as the sanction: death, exclusion, suffering, censure, loss, or duration?	The account's legal, scriptural, or moral specification of the penalty.	Naming the sanction "infinite" without identifying the relevant property.
Completion measure	What makes the sanction fully undergone or justice satisfied?	Time served, an act completed, a status changed, a debt discharged, or a justified equivalence rule.	Showing that some other property is very great or even unbounded.

The bearer question concerns substitution. A legal system may sometimes accept payment by a third party; a representative may act for a group; a tradition may assign covenantal standing to a head. None of that yet tells us whether a prison term can be completed by a payment or whether endless duration can be completed by finite suffering. Authorization to act for another and adequacy of what is done are different relations.

The penalty-kind question concerns identity. A fine, a term of exclusion, execution, public censure, and required repair can all be severe, but severity alone does not make them the same sanction. Duff's communicative account illustrates why mode matters: punishment is not generic disutility poured into interchangeable containers; its public meaning and institutional form can be part of what it is (Duff, 2001). The point does not establish one universal theory of punishment. It establishes a burden: an account that changes the relevant mode must explain why the original sanction is still satisfied rather than replaced.

The completion question concerns a rule. A hundred-dollar fine is completed when the required amount is accepted because the institution defines the obligation on a monetary scale. A ten-year term is completed through the specified time or through an authorized legal act that changes the sentence. If an executive commutes the term after one year, the prisoner's release may be lawful and merciful, but the remaining nine years were not secretly served by the executive's prestige. The sentence was altered or remitted.

These distinctions prevent two easy overstatements. Strict offender-identity conditions would rule out substitution before worth ever enters the discussion, but many PSA accounts contest precisely that restriction. Conversely, allowing a representative bearer does not settle what the representative must undergo. The Valuation Fallacy concerns the latter step: the stated measure of completion remains durational while the offered basis of completion switches to worth without an equivalence rule.

3. Incommensurability Without Overreach

3.1 No common unit is not the same as no comparison

In value theory, *incommensurability* usually means that two values lack a single common cardinal measure by which every difference can be calculated. It need not mean that the items are utterly incomparable. Raz links value pluralism to cases in which valuable options cannot be reduced to one master scale (Raz, 1986). Chang argues that some options can stand in a relation of parity even when none is simply better, worse, or equal in value (Chang, 2002). These accounts caution against a crude inference from “different dimensions” to “no rational relation of any kind.”

That correction strengthens the present argument. The paper need not prove that no institution, deity, or moral order could ever establish a relation between unlike properties. It needs only the more modest point that the relation is not supplied by magnitude words alone. If worth and duration lack a shared unit, “infinite worth” does not calculate an amount of time. An equivalence rule need not be a numerical exchange rate. It is a normative principle explaining why a performance of one kind counts as completing a requirement of another kind while preserving the features that made the requirement relevant.

3.2 Constitutive conditions set the initial burden

A condition is *constitutive* when meeting it helps make an act the relevant act rather than merely making it more impressive. Duration is constitutive of a ten-year sentence in the ordinary description “serve ten years.” Musical skill does not complete the term; intense pain does not complete it; a generous donation does not complete it. An authority may accept one of those things in exchange, but then a further rule has changed, substituted for, or discharged the requirement.

The same grammar applies to the target model. If the penalty is described as endless conscious punishment and endlessness is constitutive, satisfaction requires either endless duration or an independently justified rule explaining how another performance counts as the same completion. The worth of the substitute may be relevant to why the authority accepts the performance, why representation is appropriate, or why the act has religious significance. Those claims may support a broader account, but none by itself states the penalty’s completion rule.

This is why “three days equals eternity because the person is infinitely valuable” fails as an argument. It places value and duration on an assumed exchange schedule while offering no account of that schedule. The critique does not rest on the exact number of days, and it would apply equally to any finite interval. Finitude is the relevant feature; the conclusion is a missing bridge, not the metaphysical impossibility of every relation between worth and duration.

4. The Valuation Fallacy and the Target-Model Fork

The *Valuation Fallacy*, as used here, is the inference from the unsurpassable worth, dignity, intensity, or status of a substitute to the conclusion that the substitute has thereby been shown to satisfy a sanction whose stated completion condition is of another kind. The fallacy is conditional on the description of the sanction. A value-based obligation might be satisfied by value. A status-based restriction might be altered by a competent authority. The problem arises when the account insists on an essentially durational penalty while using non-durational worth as if it were time served.

The target-model fork

1. Either endless duration is constitutive of the penalty, or it is not.
2. If it is constitutive, a finite passion lacks that condition; worth or status alone does not supply it.
3. If it is not constitutive, the defender must identify the actual completion condition and cannot use endless duration as though it independently measures what Christ satisfied.
4. Therefore, the target account must either supply an equivalence rule, revise the penalty's identity, or distinguish literal completion from another mechanism of acceptance or release. The bare worth appeal does not complete the argument.

The fork is deductively valid as a conditional diagnostic. Its important premises remain interpretive. A defender may deny that a tradition treats endlessness as constitutive; may argue that the passion contains a qualitatively complete act rather than a quantity of suffering; or may define satisfaction by divine acceptance. Those moves deserve assessment on their own terms. They also show why a bounded critique is stronger than a universal declaration that PSA is impossible.

The comparator clarifies the result. Under an in-kind duration rule, the finite event does not match the endless requirement. Under an authorized-equivalence rule, adequacy depends on the justification of the rule. Under commutation or pardon, the requirement is altered or released. Under representation, the bearer relation changes, but the penalty-kind and completion questions remain. The target inference is not rescued until one of these additional accounts does real work. This is the categorical failure named in the title: the argument crosses from one kind of property to another without stating the rule that carries the conclusion. It is a failure of the inference as presented, not proof that every bridge is impossible.

5. Applying the Completion-Rule Test to Penal Substitution

Defenses of PSA often make claims more careful than the target package. Jeffery, Ovey, and Sach defend Christ as bearing divine judgment in the place of sinners and situate that claim within a broader scriptural and doctrinal account (Jeffery et al., 2007). Craig explicitly engages the coherence and justification of penal substitution using concepts from law, representation, punishment, and pardon (Craig, 2020). Neither work should be reduced to a sentence about valuable days purchasing endless years.

This matters dialectically. If a defender says the passion is satisfactory because the Christian deity accepts a representative, legally imputed punishment, then the central question is whether that acceptance is just and what makes the punishment equivalent. If the defender says Christ experiences the full qualitative essence of divine judgment, then the relevant penalty may be a completed act or status rather than endless duration. If the defender says the passion grounds a pardon, the account can distinguish the basis for mercy from literal completion of every element of the threatened sentence.

Each option can avoid the simplest Valuation Fallacy, but none licenses the original four-claim package without further work. Representation addresses who acts for whom. Divine acceptance identifies an authoritative decision. Qualitative completion revises the measure. Pardon or commutation releases a liability. These are meaningful mechanisms, not synonyms for worth becoming duration.

The language of satisfaction is therefore dangerous when it floats between models. In one paragraph it may mean that the precise penalty has been undergone. In another it may mean that justice has an adequate reason to release the liable parties. In another it may mean that covenantal or relational aims have been fulfilled. A valid argument must hold the meaning fixed or explain the transition.

The test bears most directly on the following apologetic shortcut: sinners allegedly owe endless conscious punishment; Christ did not undergo an endless interval; nevertheless the debt is fully paid because an infinitely

valuable person suffered. That shortcut uses duration to define and magnify the debt, then abandons duration when explaining payment. The shift may point toward a defensible account, but it does not yet explain why the same obligation has been completed.

6. The Strongest Replies and Their Costs

The following replies are not evasions merely because they preserve theological possibility. Each identifies a path a serious account might take. The ledger records whether the reply changes the bearer, the penalty, the completion rule, or the mode of release; it then identifies what the path must still establish.

Reply	What it preserves	What it changes or adds	Remaining burden
Endlessness is consequential	PSA and endless postmortem loss can both remain.	Duration is no longer the constitutive measure Christ had to undergo.	Specify the actual penalty Christ satisfies and why it is equivalent.
Infinite intensity or dignity	The passion can be uniquely weighty and religiously decisive.	Introduces quality or status as the measure of adequacy.	Justify why that measure satisfies, rather than replaces, a durational condition.
Representative union or headship	A substitute can stand for others within the account.	Supplies a bearer relation.	Show what the representative undergoes and why it completes the specified sanction.
Divine pardon or commutation	Release can be lawful, principled, and responsive to the passion.	Treats the threatened liability as remitted or altered.	Avoid describing release as literal time served if the time was not undergone.
Qualitatively complete judgment	Christ may undergo the full kind of judgment in a finite act.	Defines completion by act-type or quality, not endless duration.	Explain why endless punishment remains the sinners' penalty if duration is not essential.
Mystery or special divine rule	Leaves logical space for a relation beyond present understanding.	Posits a bridge rather than deriving it from worth.	Mystery preserves possibility but does not publicly establish satisfaction.

6.1 The penalty is death or judgment, not endless duration

This is the cleanest reply. The defender can say that Christ bears death, condemnation, curse, or divine judgment; the endless condition of the unredeemed is a consequence of remaining alienated, not a unit of punishment that the substitute must reproduce. If so, the target's second commitment is false. The Valuation Fallacy no longer applies in its durational form.

The cost is not necessarily theological; it is argumentative. The defender must stop using the endless duration of hell as the quantity that makes the debt enormous while using a different measure when the debt is declared paid. The account should name the finite or qualitative act that constitutes satisfaction.

6.2 The passion has infinite intensity, dignity, or moral value

This reply grants that duration differs while proposing a compensating property. It may have a place in a moral or theological account. A uniquely dignified agent's voluntary act could carry representative, expressive, or covenantal significance unavailable to an ordinary act. But "infinite" still does not generate the bridge. The reply must explain why the governing norm permits intensity, dignity, or value to replace duration.

Once that norm is supplied, the view is not merely a magnitude claim. It is an account of what justice values. That is progress. It also makes the dispute visible: critics can ask whether the proposed norm respects the

stated penalty-kind, whether it treats persons as commensurable assets, and whether acceptance by the authority amounts to satisfaction or commutation.

6.3 Union, imputation, or federal headship authorizes the substitute

Representative theories answer a real objection: why may one person's act count for others? If a covenantal head truly represents a people, identity need not be transferred as if the persons were numerically identical. The act can be theirs in a juridical or corporate sense.

Yet representation does not make every performance adequate. An authorized agent who pays half a fine has the right bearer relation but has not completed the amount unless another rule covers the remainder. Likewise, headship can explain why Christ may act for sinners while leaving open whether the act matches the specified sanction. Bearer and completion must not be collapsed.

6.4 Pardon and commutation are not arbitrary

Mercy need not be a failure of justice. Murphy and Hampton's analyses distinguish forgiveness, mercy, resentment, and retributive judgment without assuming that every release is a denial that wrongdoing matters (Murphy & Hampton, 1988). A legal or theological authority can have principled reasons to remit a liability, especially where public censure, repentance, repair, reconciliation, or another good is achieved.

The terminology should then be exact. Commutation changes a sanction; pardon releases liability; satisfaction completes the relevant requirement. An account may connect them—for example, the passion may furnish the morally adequate basis for pardon—without claiming that a finite event literally instantiates endless duration. That model can be coherent while conceding the paper's narrow point.

6.5 Atonement need not be exhausted by penal satisfaction

Christian atonement has been analyzed through participation, union, healing, victory, sacrifice, reconciliation, moral transformation, and other motifs. Crisp surveys the range while developing a constructive account, and Stump offers a love- and union-centered interpretation directed at guilt and shame (Crisp, 2020; Stump, 2018). Murphy argues that normative obstacles remain important even within an account centered on divine holiness (Murphy, 2021). These differences show that rejecting a value-for-duration inference does not leave Christian theology without conceptual resources.

The constructive lesson is not “choose any metaphor.” It is to let each mechanism state its own success conditions. A healing account should explain what is healed. A reconciliatory account should explain how relationship is restored. A penal account should explain what punishment is borne and what completes it. A plural account should explain how its elements relate rather than allowing one vocabulary to borrow the appearance of precision from another.

7. What the Completion-Rule Test Establishes

The paper establishes a conditional result with practical bite: an account must use one completion rule on both sides of the transaction or explain the principled change.

First, the target inference is invalid without a bridge. From “the substitute has infinite worth” and “the penalty is endless duration,” no conclusion about completed duration follows. The premises concern different properties.

Second, the bridge cannot be hidden inside the word *satisfies*. If satisfaction means meeting the constitutive conditions, the duration problem remains. If it means giving the authority adequate reason to release the liable party, the account has moved toward pardon, commutation, or a broader normative settlement. That move may be defensible, but it should be stated.

Third, the fork disciplines both critic and defender. The critic should not claim to have refuted every PSA model merely because one inference is unsupported. The defender should not answer a challenge to literal penal completion with a different claim about religious significance and then pretend nothing changed.

Fourth, the argument exposes a double standard in apologetic rhetoric. Endless duration is often used when magnifying what sinners deserve, while the duration condition disappears when measuring what the substitute undergoes. A consistent account must apply one satisfaction rule to both sides or explain the principled difference.

The result does not establish that theism is false, that a Christian God-concept cannot forgive, that no representative punishment is possible, or that endless punishment is independently incoherent. It shows that one familiar route from infinite worth to durational satisfaction is unsupported. A live alternative remains wherever a defender provides a coherent account of penalty identity, representation, equivalence, or release.

8. Conclusion

If an alleged penalty is essentially endless in duration, infinite worth alone does not establish that a finite passion completes it. Worth may explain significance or acceptance; it is not itself time undergone or an equivalence rule. The target account must therefore deny that duration is constitutive, justify why another performance counts as equivalent, or distinguish literal satisfaction from qualitative completion, pardon, commutation, or a non-penal account. Representation may authorize the substitute, but it still needs an account of completion or release.

The central demand is consistency: an account cannot make the penalty durational when measuring what sinners owe and non-durational when measuring payment unless it explains the change. This does not refute every theory of penal substitution. It removes an illicit shortcut. Bearer, penalty-kind, and completion are distinct; worth and duration lack an automatic common measure; and every live reply supplies a relation or revises the target. Readers should ask: what exactly completes the penalty, and where was that condition met or lawfully changed?

A. Formal Reconstruction of the Central Thesis

A.1 Plain-language reconstruction

1. **Premise 1.** If an account defines endless duration as a constitutive condition of a penalty, defines the substitute's passion as finite, and offers worth or status without an independently justified equivalence rule, then that offer does not establish satisfaction of the penalty.
2. **Premise 2.** The target PSA account has exactly those features: an essentially endless penalty, a finite passion, and worth or status offered without a defended equivalence rule.
3. **Conclusion.** Therefore, the target PSA account does not establish that the finite passion satisfies the essentially endless penalty.

The inference is valid by modus ponens. Premise 1 expresses the difference between satisfying a requirement and offering another property; Premise 2 is interpretive when applied to a real defense, so the critic must show that it retains the target conjunction. Denying constitutive duration, supplying an equivalence rule, or identifying another completion or release mechanism blocks Premise 2 and confirms the scope limit.

A.2 Symbols

- P : the target account makes endless duration constitutive of the penalty.
- Q : the substitute's passion is finite in duration.
- R : worth or status is offered without an independently justified equivalence rule.
- C : the target account establishes that the passion satisfies the penalty.

A.3 Symbolic form

$$\begin{aligned} (P \wedge Q \wedge R) &\rightarrow \neg C \\ P \wedge Q \wedge R \\ \therefore \neg C \end{aligned}$$

The form is valid by modus ponens. Here $\neg C$ means only that the target account fails to establish satisfaction under the stated conditions, not that every PSA account or atonement mechanism fails.

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